Løsningsforslag Mekanikk eksamen 13.12.2006

Oppgave 1

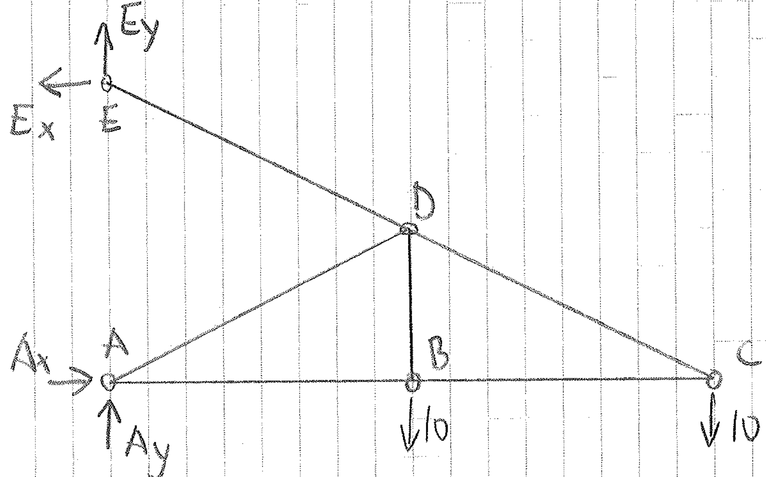
(a) Antall knutepunkt : $k = 5$

Ant. likevektslikninger : $2k = 10$

Ant. reaksjonskrefter : $r = 4$

Ant. staver : $s = 6$

$$2k = r + s \quad \text{dvs statisk likevekt}$$



Hele konstr :

$\sum \overset{\curvearrowright}{M}_A = 0$ gir $10 \cdot 4 + 10 \cdot 8 - E_x \cdot 4 = 0$ $\Rightarrow$ $\underline{E_x = 30 kN}$

DE :

$\sum \overset{\curvearrowright}{M}_D = 0$ gir $E_y \cdot 4 - E_x \cdot 2 = 0$ $\Rightarrow$ $\underline{E_y = 15 kN}$

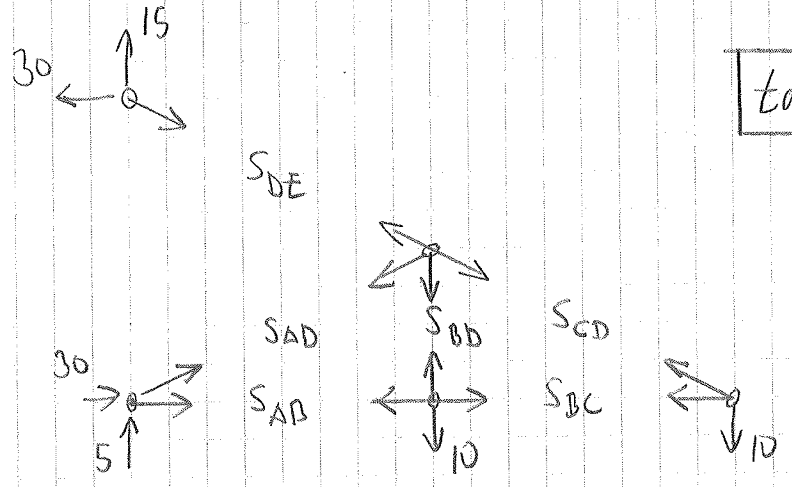
Hele konstr :

$\uparrow \sum F_y = 0$ gir $A_y + 15 - 10 - 10 = 0$ $\Rightarrow$ $\underline{A_y = 5 kN}$

$\overset{\rightarrow}{\sum F_x} = 0$ gir $\underline{A_x = 30 kN}$

(b) Frilegeme-diagram:

$\tan\alpha = \frac{2}{4} \Rightarrow \alpha = 26,57°$



KP. A: $\sum F_y = 0$ gir $5 + S_{AD} \sin 26,57° = 0$ $\Rightarrow$ $S_{AD} = -11,18 kN$

$\sum F_x = 0$ gir $30 + S_{AB} + (-11,18)\cos 26,57° = 0$ $\Rightarrow$ $S_{AB} = -20 kN$

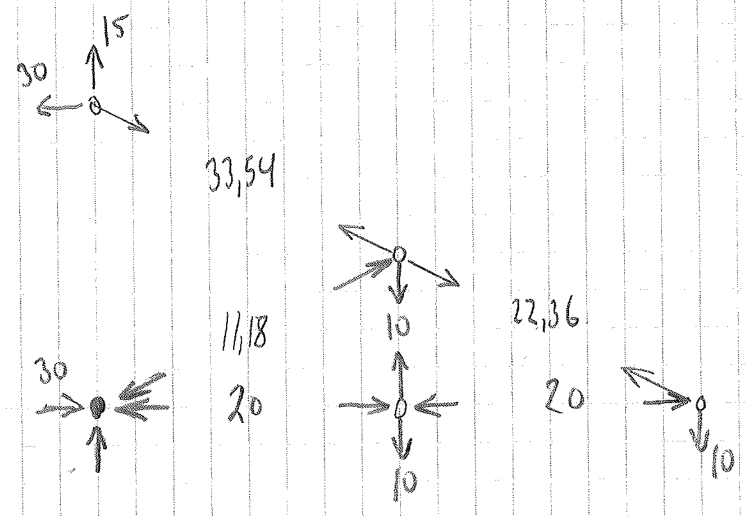
KP. E: $S_{DE} = \sqrt{15^2 + 30^2} = 33,54 kN$

KP. B: $S_{BC} = S_{AB} = -20 kN$

$S_{BD} = 10 kN$

KP. C: $\sum F_y = 0$ gir $S_{CD} \sin 26,57° - 10 = 0$ $\Rightarrow$ $S_{CD} = 22,36 kN$

Belastningsdiagram:



(c)

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(32^2 - 20^2) = 490\ mm^2$$

$$l = \sqrt{2^2 + 4^2} = 4{,}47\ m$$

$$F = 33{,}54\ kN$$

$$\underline{\underline{\Delta l}} = \frac{Fl}{EA} = \frac{33540 \cdot 4472}{206000 \cdot 490} = \underline{\underline{1{,}49\ mm}}$$

(d)

Største normalspenninger:

$$\sigma_N = \frac{F}{A} = \frac{33540}{490} = 68\ MPa < 160\ MPa \quad \text{dvs ok}$$

Knekning:

$$I_0 = \frac{\pi}{64}(32^4 - 20^4) = 43{,}6 \cdot 10^3\ mm^4 \quad , \quad i = \sqrt{\frac{I_0}{A}} = 9{,}43\ mm$$

Sjekker stav AD:

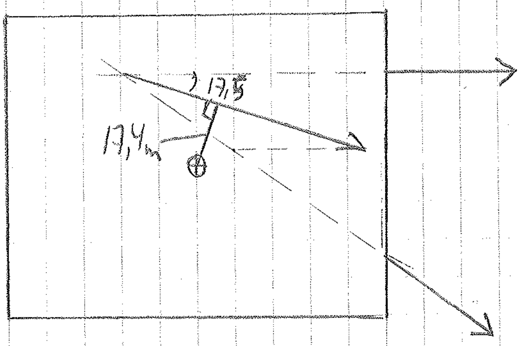
$$\lambda = \frac{4472}{9{,}43} = 455 \quad \text{dvs. elastisk knekning}$$

$$\underline{F_K} = \frac{\pi^2 E I_0}{l_K^2} = \frac{\pi^2 \cdot 206000 \cdot 43600}{4472^2} = \underline{4{,}43\ kN}$$

$S_{AD} > F_K$ dvs vi får elastisk knekning

Oppgave 2

(a)



$F_{Rx} = 700 + 700\cos 35^\circ = 1273{,}4\,kN$

$F_{Ry} = -700 \sin 35^\circ = -401{,}5\,kN$

$\underline{F_R} = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \underline{1335\,kN}$

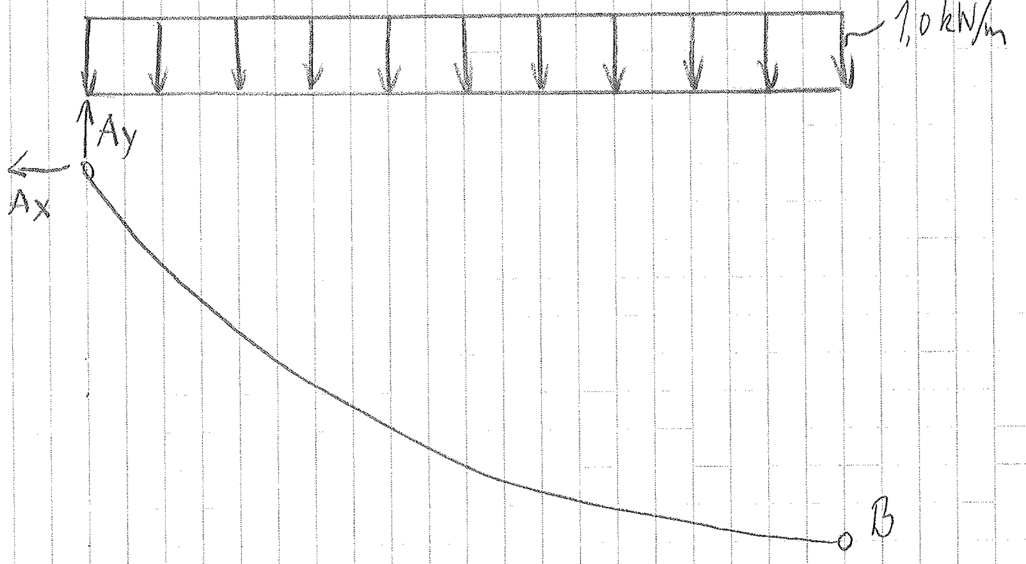
$\tan \alpha_R = \frac{F_{Ry}}{F_{Rx}} \Rightarrow \underline{\alpha_R = -17{,}5^\circ}$

Moment om senterpunktet:

$\curvearrowright M_R = 25 \cdot 700 - 25 \cdot 700 \cdot \cos 35^\circ + 50 \cdot 700 \sin 35^\circ = 23240\,kNm$

$\underline{a} = \frac{M_R}{F_R} = \frac{23240}{1335} = \underline{17{,}4\,m}$

(b)

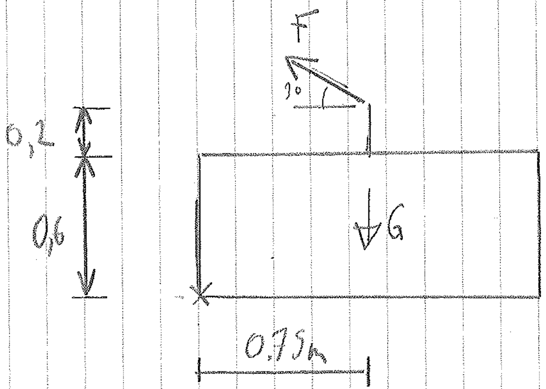


$\sum F_y = 0$ gir $A_y = 1{,}0 \cdot 200 = \underline{200\,kN}$

$\sum \hat{M}_B = 0$ gir $A_y \cdot 200 - A_x \cdot 100 - 1{,}0 \cdot 200 \cdot 100 = 0 \Rightarrow \underline{A_x = 200\,kN}$

$\underline{A_R = 283\,kN}$

(c) Loddets tyngde : $G = \rho g h \frac{\pi}{4} d^2 = 2{,}3 \cdot 9{,}81 \cdot 0{,}6 \cdot \frac{\pi}{4} \cdot 1{,}5^2 = 23{,}9\,kN$



Veltemoment :

$M_v = 0{,}8 \cdot 15 \cos 30° + 0{,}75 \cdot 15 \cdot \sin 30° = \underline{16{,}0\,kNm}$

Stabiliseringsmoment :

$\underline{M_s} = 23{,}9 \cdot 0{,}75 = \underline{17{,}9\,kNm}$

$M_s > M_v$ dvs loddet vil ikke velte

Normalkraft på underlaget : $F_N = G - F \sin 30° = 23{,}9 - 15 \sin 30° = 16{,}4\,kN$

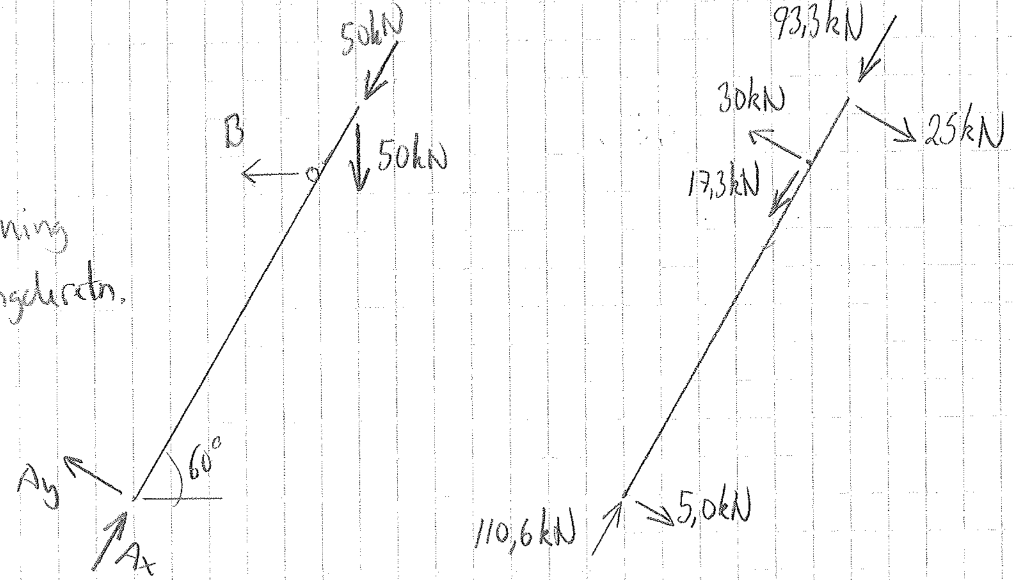
Største friksjonskraft : $F_F = \mu F_N = 6{,}56\,kN$

$F_x = 15 \cos 30° = 13{,}0\,kN$ $F_x > F_F$, dvs loddet vil gli

Oppgave 3

(a)

NB: Velger x-retning i bommens lengderetn.



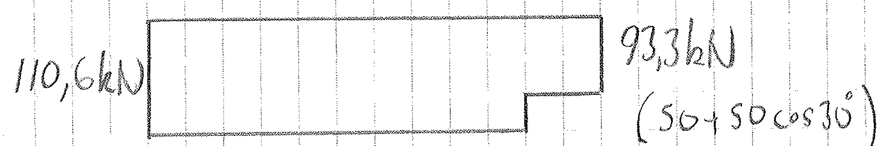
$\Sigma \hat{M}_A = 0$ gir $50 \cdot 6 \cdot \cos 60 - B \cdot 5 \cdot \sin 60° = 0 \Rightarrow \underline{B = 34{,}64 kN}$

$\Sigma F_x = 0$ gir $A_x - 50 - 50 \cos 30° - 34{,}64 \cdot \cos 60° = 0 \Rightarrow A_x = 110{,}6 kN$

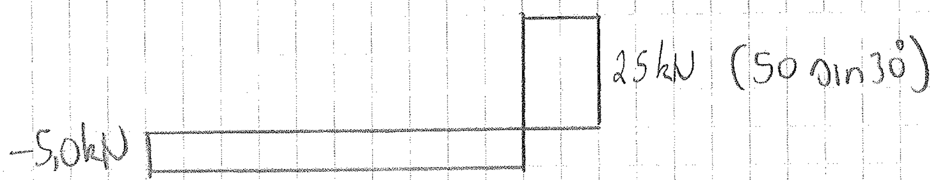
$\Sigma F_y = 0$ gir $A_y + 34{,}64 \cdot \sin 60° - 50 \sin 30° = 0 \Rightarrow A_y = -5{,}0 kN$

(b)

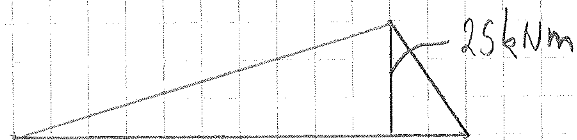
N-diagram:



V-diagram:



M-diagram:



(c)

$$\sigma_{max} = \frac{M}{W_{min}} \Rightarrow W_{min} = \frac{25 \cdot 10^6}{160} = 156\,cm^3$$

IPE200 har motstandsmoment $W_x = 194\,cm^3$
$A = 28,5\,cm^2$

$$\sigma_B = \frac{M}{W} = \frac{25 \cdot 10^6}{194 \cdot 10^3} = 128,9\,MPa$$

$$\sigma_A = \frac{F}{A} = \frac{110,6 \cdot 10^3}{2850} = 38,8\,MPa$$

Største normalspenninger ; $\sigma_N = \sigma_A + \sigma_B = 167,7\,MPa$ dvs ikke ok.

Prøver IPE220 ; $W_x = 252\,cm^3$, $A = 33,4\,cm^2$

$\sigma_B = \frac{25 \cdot 10^6}{252 \cdot 10^3} = 99,2\,MPa$ $\quad \sigma_A = \frac{110,6 \cdot 10^3}{3340} = 33,1$ $\quad \underline{\sigma_N = 132\,MPa}$

Velger derfor IPE220

(d) Bjelken vil knekke ut sideveis, dermed har ståltauet i B ingen innvirkning
velger derfor $l_k = 6,0\,m$

IPE 220 : $I_0 = 205\,cm^4$

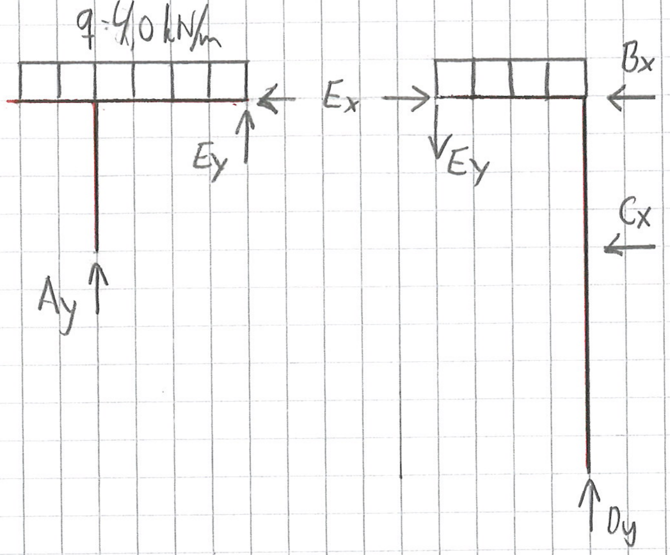
$$F_k = \frac{\pi^2 E I_0}{n\, l_k^2} = \frac{\pi^2 \cdot 206000 \cdot 2050000}{1,4 \cdot 6000^2} = 82,7\,kN < 110,6 \text{ dvs ikke ok}$$

IPE 240 ; $I_0 = 284\,cm^4$, $i = 2,69\,cm$

$F_k = 114,6\,kN > 110,6$ dvs ok, $\quad \lambda = \frac{l_k}{i} = \frac{6000}{26,9} = 223$ dvs elastisk knekning

Oppgave 4

(a)



Antall elementer : $e=2$

Ant. likninger : $3e=6$

Ant. ukjente : $r=6$

$3e=r$ dvs statisk bestemt.

Venstre del:

$\Sigma F_x = 0$ gir $\underline{E_x = 0}$

$\Sigma \hat{M}_A = 0$ gir $4 \cdot 3 \cdot 0{,}5 - E_y \cdot 2 = 0 \Rightarrow \underline{E_y = 3{,}0 \text{ kN}}$

$\Sigma F_y = 0$ gir $A_y + 3{,}0 - 4 \cdot 3 = 0 \Rightarrow \underline{A_y = 9{,}0 \text{ kN}}$

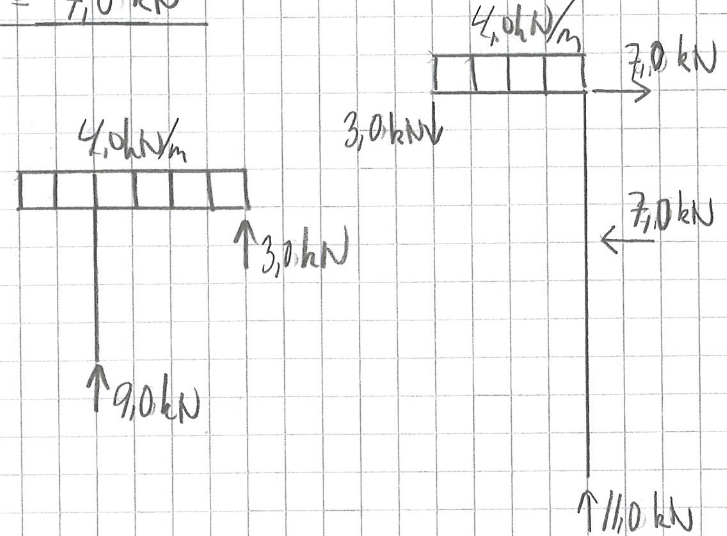
Høyre del:

$\Sigma F_y = 0$ gir $D_y - 3{,}0 - 4 \cdot 2 = 0 \Rightarrow \underline{D_y = 11{,}0 \text{ kN}}$

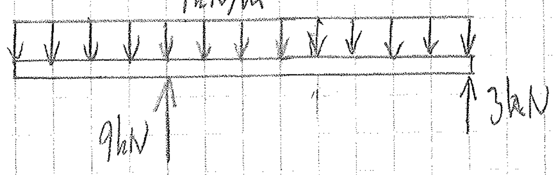
$\Sigma \hat{M}_B = 0$ gir $-3{,}0 \cdot 2 - 4 \cdot 2 \cdot 1 + C_x \cdot 2 = 0 \Rightarrow \underline{C_x = 7{,}0 \text{ kN}}$

$\Sigma F_x = 0$ gir $\underline{B_x = -7{,}0 \text{ kN}}$

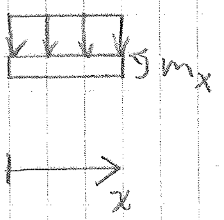
Belastningsdiagram:



(b) Venstre del:



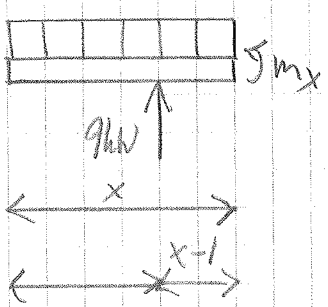
$x \in [0,1]$:



$M_i = M_y$ gir $m_x = -4x \cdot \frac{x}{2} = -2x^2$

$x = 1m$ gir $m_x = -2kNm$

$x \in [1,2]$:

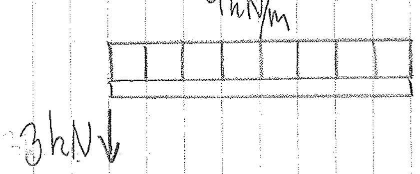


$M_i = M_y$ gir $m_x = -2x^2 + 9(x-1)$
$= -2x^2 + 9x - 9$

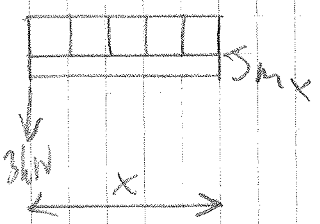
$m_x' = -4x + 9$ $\quad$ $m_x' = 0$ gir $x = 2{,}25m$

$x = 2.25m$ gir $m_x = -2(2.25)^2 + 9 \cdot 2.25 - 9 = 1{,}125 kNm$

Høyre del:



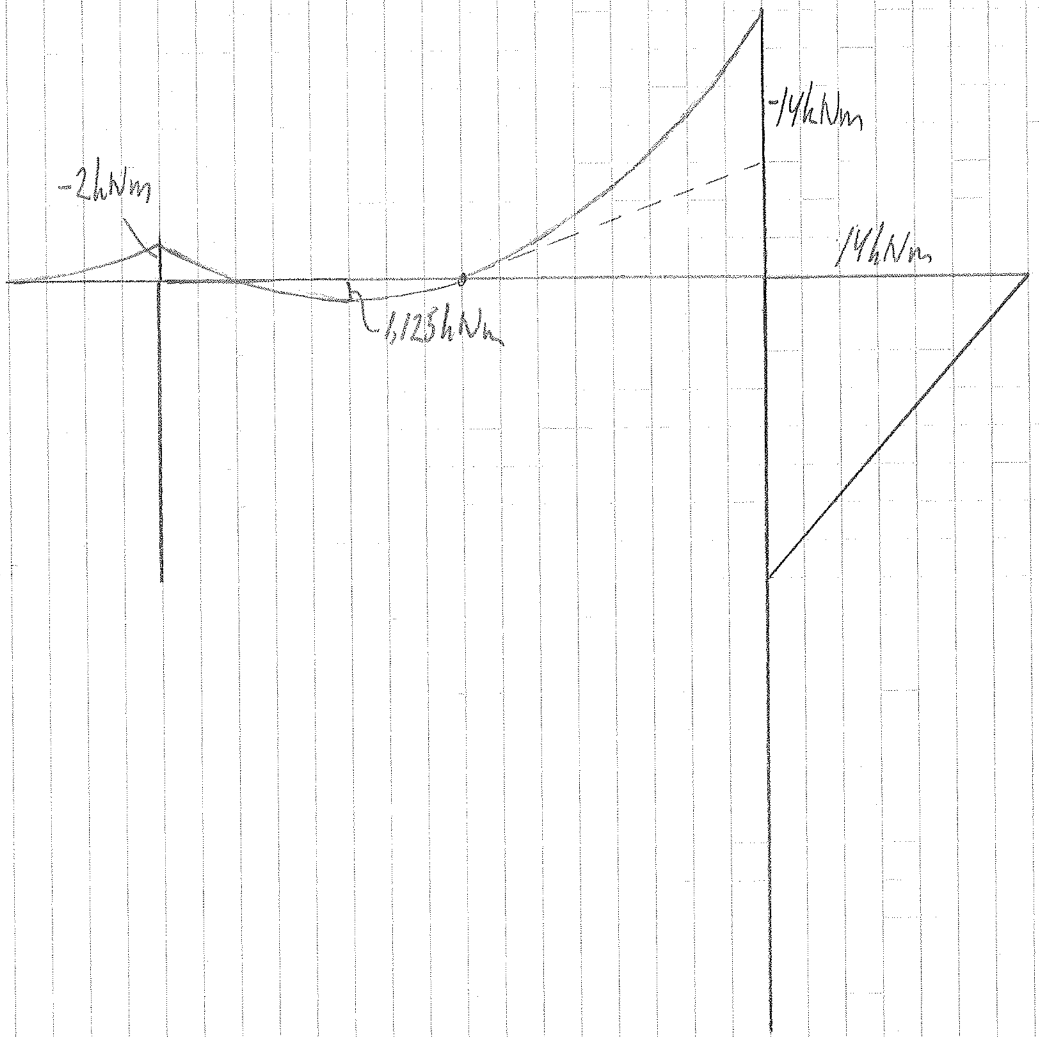
$x \in [0,2]$



$M_i = M_y$ gir $m_x = -2x^2 - 3x$

$x = 2$ gir $m_x = -14kNm$

Bøyemomentdiagram



(c)

$$W_{min} = \frac{M}{\sigma_{max}} = \frac{14,0 \cdot 10^6}{160} = 87,5\,cm^3$$

IPE160 har $W_x = 109\,cm^3$ og $A = 20,1\,cm^2$

$$\sigma_A = \frac{11 \cdot 10^3}{2010} = 5,47\,MPa$$

$$\sigma_B = \frac{14,0 \cdot 10^6}{109 \cdot 10^3} = 128,44\,MPa$$

$\sigma_{N,max} = \sigma_A + \sigma_B = 133,9\,MPa$ dvs IPE160 er ok