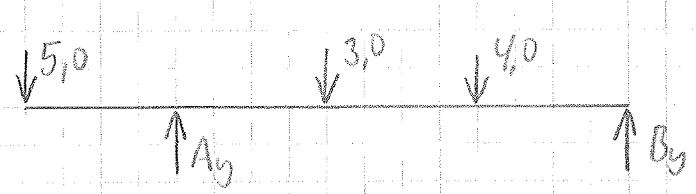
Eksamen Mekanikk 2006B, kontinuasjon 27.02.2007 ①

LØSNING

## Oppgave 1

a)



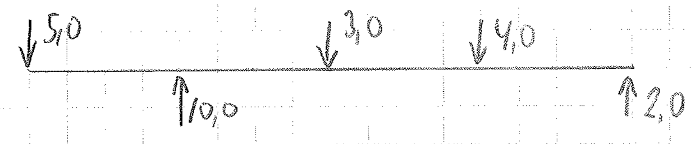

$$\overset{\curvearrowleft}{\sum M_A} = 0 \quad \text{gir} \quad -5\cdot 2 + 3\cdot 2 + 4\cdot 4 - B_y\cdot 6 = 0$$

$$\Rightarrow \quad \underline{B_y = 2{,}0\,kN}$$

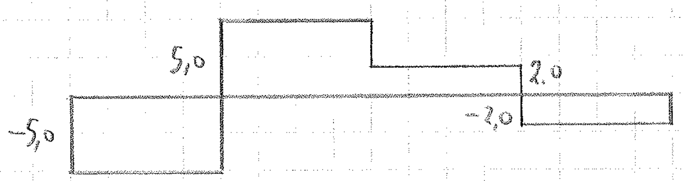
$$\uparrow \sum F_y = 0 \quad \text{gir} \quad A_y + B_y - 5{,}0 - 3{,}0 - 4{,}0 = 0$$

$$\Rightarrow \quad \underline{A_y = 10{,}0\,kN}$$

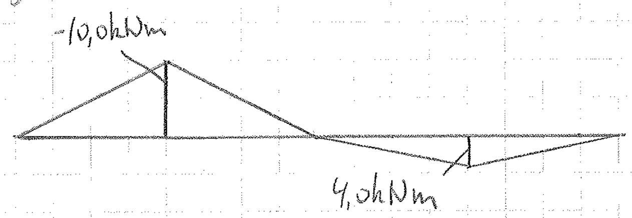
Belastningsdiagram:




(b) Skjærkraftdiagram:




Bøyemomentdiagram:



(c) Bøyespenningsformelen:

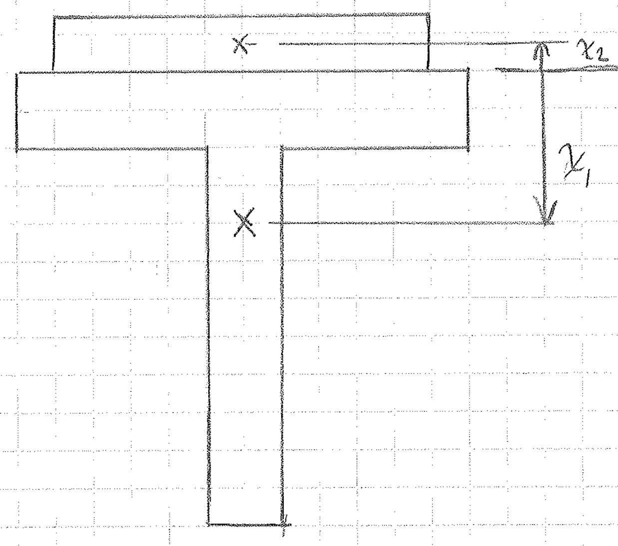
$$\sigma_b = \frac{M_b}{W}$$

$$\Rightarrow \underline{W_{min}} = \frac{M_b}{\sigma_{b,max}} = \frac{10 \cdot 10^6}{160} = 62500\,mm^3 = \underline{62{,}5\,cm^3}$$

Profilet T140 har $W_x = 64{,}7\,cm^3$ dvs ok

(d) Tverrsnittsdata for T120:

$A = 2960\,mm^2$ , $I_x = 366 \cdot 10^4\,mm^4$ , $d = 32{,}8\,mm$




Felles nøytralakse:

$$\underline{\bar{X}} = \frac{A_1 x_1 - A_2 x_2}{A_{TOT}} = \frac{2960 \cdot 32{,}8 - 1000 \cdot 5}{3960} = \underline{23{,}3\,mm}$$

Totalt arealmoment:

$$\underline{I} = 366 \cdot 10^4 + 2960 \cdot (\underbrace{32{,}8 - 23{,}3}_{9{,}5})^2 + \frac{1}{12} 100 \cdot 10^3 + 1000 \cdot (\underbrace{23{,}3 + 5{,}0}_{28{,}3})^2$$

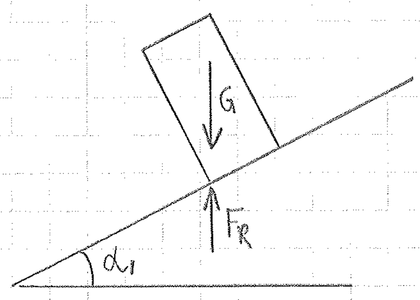
$$= 366 \cdot 10^4 + 26{,}7 \cdot 10^4 + 0{,}83 \cdot 10^4 + 80{,}1 \cdot 10^4 = \underline{4{,}74 \cdot 10^6\,mm^4}$$

Største bøyespenninger:

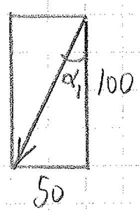
$$\underline{\sigma_b} = \frac{M}{I} \cdot y = \frac{10 \cdot 10^6}{4{,}74 \cdot 10^6} \cdot (\underbrace{120 - 23{,}3}_{96{,}7}) = \underline{204 \frac{N}{mm^2}}$$

## Oppgave 2

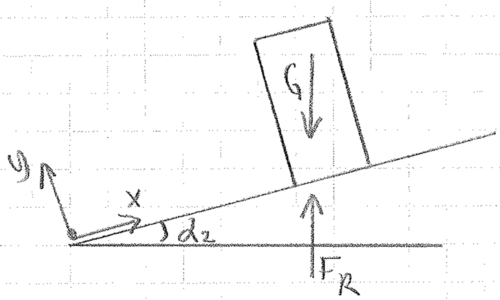
(a)




Klossen velter når vinkelen blir så stor at tyngden G "peker" mot hjørnet.




$\tan\alpha_1 = \frac{50}{100} \Rightarrow \underline{\underline{\alpha_1 = 26{,}6^\circ}}$

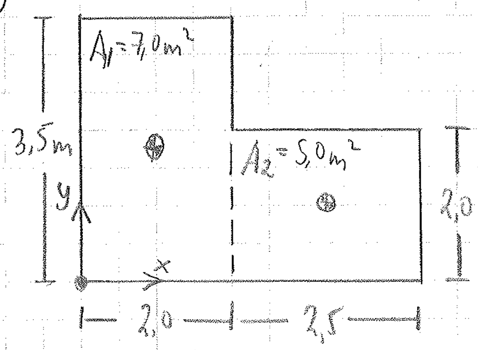



Klossen glir når

$\tan\alpha_2 = \mu$

$\mu = 0{,}35$ gir $\underline{\underline{\alpha_2 = 19{,}3^\circ}}$

(b)




$\underline{\underline{\bar{x}}} = \frac{A_1x_1 + A_2x_2}{A_{TOT}} = \frac{7\cdot 1 + 5\cdot 3{,}25}{12} = \underline{\underline{1{,}94\,m}}$

$\underline{\underline{\bar{y}}} = \frac{A_1y_1 + A_2y_2}{A_{TOT}} = \frac{7\cdot 1{,}75 + 5\cdot 1}{12} = \underline{\underline{1{,}44\,m}}$

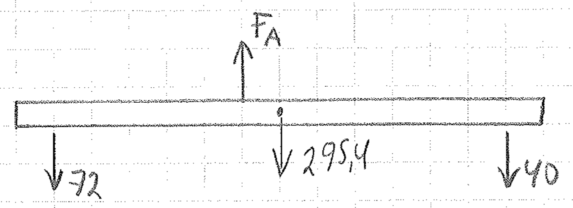
Platas tyngde: $\rho_{stål} = 7850\,kg/m^3$

$G = 12{,}0\cdot 0{,}050\cdot 7{,}85\cdot 9{,}81 = \underline{\underline{46{,}2\,kN}}$

(c)

IPE300: $g = 42{,}2\,\text{kg/m}$

$$G = 42{,}2 \cdot 7{,}0 = 295{,}4\,\text{kg}$$



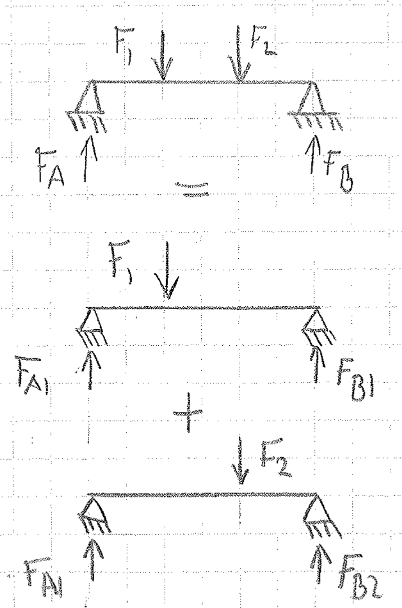

$$\sum \overset{\curvearrowleft}{M_A} = 0 \quad \text{gir} \quad -72 \cdot 2{,}5 + 295{,}4 \cdot 0{,}5 + 40 \cdot 3{,}5 + a \cdot 60 = 0$$

$$\underline{\underline{a = -1{,}80\,\text{m}}}$$

Bjelken kommer i likevekt hvis 60kg henges 1,8m til venstre for A

(d) For en konstruksjon som er utsatt for flere laster kan man beregne virkningen fra hver av lastene, hver for seg.
Den totale virkningen vil være lik summen av virkningene fra den enkelte last. Med virkning mener vi reaksjonskrefter, indre spenninger og deformasjoner.
Prinsippet forutsetter små deformasjoner.

Eks:

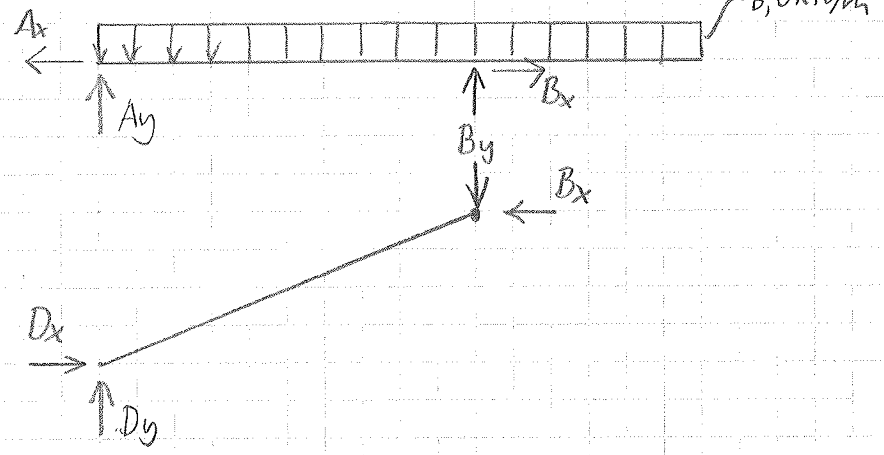



$$F_A = F_{A1} + F_{A2}$$

$$F_B = F_{B1} + F_{B2}$$

## Oppgave 3

(a) FLD:




**Element AC:**

$\sum \overset{\curvearrowleft}{M}_A = 0$ gir $6{,}0 \cdot 4 \cdot 2 - B_y \cdot 2{,}5 = 0 \Rightarrow \underline{B_y = 19{,}2\,kN}$

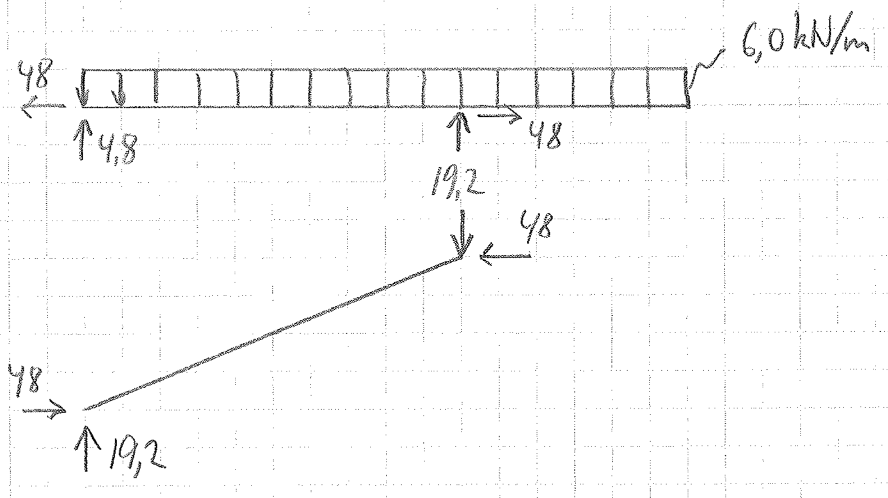
$\uparrow \sum F_y = 0$ gir $A_y + 19{,}2 - 6{,}0 \cdot 4 = 0 \Rightarrow \underline{A_y = 4{,}8\,kN}$

**Element BD:**

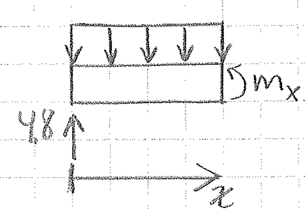
$\sum \overset{\curvearrowleft}{M}_D = 0$ gir $19{,}2 \cdot 2{,}5 - B_x \cdot 1{,}0 = 0 \Rightarrow \underline{B_x = 48\,kN}$

Ser uten videre at $D_x = B_x$ $D_y = B_y$ og $A_x = B_x$

**Belastningsdiagram:**



(b) Intervall AB:




$m_i = M_y$ gir

$m_x = 4{,}8x - 6{,}0 \cdot x \cdot \frac{x}{2} = 4{,}8x - 3x^2$

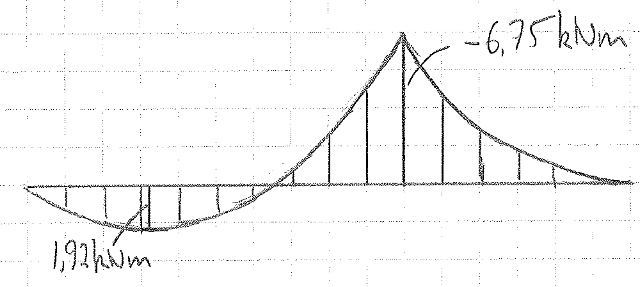
Finner bunnpunkt:

$m_x' = 4{,}8 - 6x = 0 \Rightarrow x = 0{,}8\,m$

$x = 0{,}8$ gir $m_x = 1{,}92\,kNm$

$x = 2{,}5$ gir $m_x = -6{,}75\,kNm$

Bøyemomentdiagram:




(c) $\underline{W_{min}} = \frac{M_B}{\sigma_{min}} = \frac{6{,}75 \cdot 10^6}{160} = 42188\,mm^3 = \underline{42{,}2\,cm^3}$

IPE120 har $W_x = 53{,}0\,cm^3$ og $A = 13{,}2\,cm^2$

Sjekker spenningene rett til venstre for B hvor vi også har aksiallast på 48kN

$$\left.\begin{aligned} \sigma_B &= \frac{M}{W_x} = \frac{6{,}75 \cdot 10^6}{53{,}0 \cdot 10^3} = 127{,}4\,\frac{N}{mm^2} \\ \sigma_A &= \frac{F}{A} = \frac{48 \cdot 10^3}{1320} = 36{,}4\,\frac{N}{mm^2} \end{aligned}\right\} \underline{\sigma_N} = \sigma_A + \sigma_B = \underline{164\,\frac{N}{mm^2}} > 160$$

Går derfor opp en dimensjon og velger IPE140

(d)

Knekklengde $l_k = \sqrt{2{,}5^2 + 1{,}0^2} = 2{,}69\,m$

Treghetsradius: $i = 1{,}65\,cm$

Slankhet: $\lambda = \frac{l_k}{i} = \frac{2690}{16{,}5} = 163 > 105$ dvs elastisk knekning

Arealmoment (minste) $I_o = 44{,}9\,cm^4$

Eulerlast

$$\underline{F_E} = \frac{\pi^2 E I_o}{l_k^2} = \frac{\pi^2\, 206000 \cdot 449000}{2690^2} = \underline{126\,kN}$$

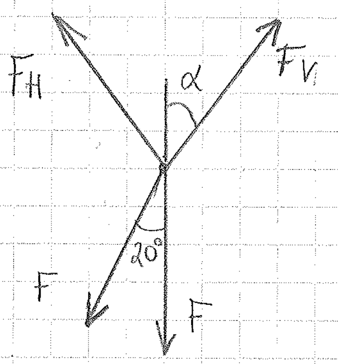
Aksiallast i stag BD: $\underline{F} = \sqrt{48^2 + 19{,}2^2} = \underline{51{,}7\,kN}$

Sikkerhetsfaktor: $\frac{F_E}{F} = \frac{126}{51{,}7} = 2{,}4 > 2{,}0$ dvs ok

## Oppgave 4

$$F = 60\,kg \cdot 9{,}81 \frac{N}{kg} = 589\,N$$

Ser på kreftene som virker på toppunktet:




$$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36{,}9^\circ$$

$$\cos\alpha = 0{,}8, \quad \sin\alpha = 0{,}6$$

$\Sigma F_x = 0$ gir $\quad F_V \sin\alpha - F_H \sin\alpha - F \sin 20 = 0$

$$F_V = \frac{F_H \cdot 0{,}6 + 589 \cdot \sin 20}{0{,}6} = F_H + 335{,}7 \qquad (1)$$

$\Sigma F_y = 0$ gir $\quad F_V \cdot \cos\alpha + F_H \cos\alpha - F - F\cos 20 = 0$

$$F_V + F_H = \frac{589(1 + \cos 20)}{0{,}8} = 1428 \qquad (2)$$

Setter (1) inn i (2):

$$F_H + 335{,}7 + F_H = 1428$$

$$\underline{\underline{F_H = 546\,kN}}$$

(1):

$$\underline{F_V} = 546 + 335{,}7 = \underline{\underline{882\,kN}}$$