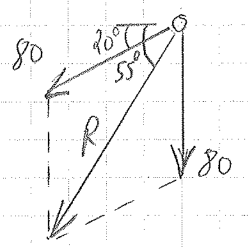
Eksamen Mekanikk 6/12-2007, Løsning

Oppg. 1

a)



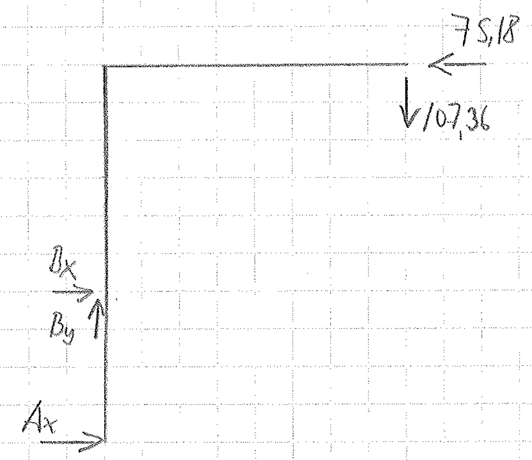
$D_y = 80 + 80\sin 20° = 107{,}36\,kN$

$D_x = 80\cos 20° = 75{,}18\,kN$

$\underline{\underline{D}} = \sqrt{D_x^2 + D_y^2} = \underline{\underline{131{,}1\,kN}}$

$\tan\alpha = \frac{D_y}{D_x} = 1{,}43 \Rightarrow \underline{\underline{\alpha = 55{,}0°}}$

b) Fritt-Legeme-diagram:



$\sum F_y = 0$ gir $\underline{\underline{B_y = 107{,}36\,kN}}$

$\sum \overset{\curvearrowleft}{M_B} = 0$ gir

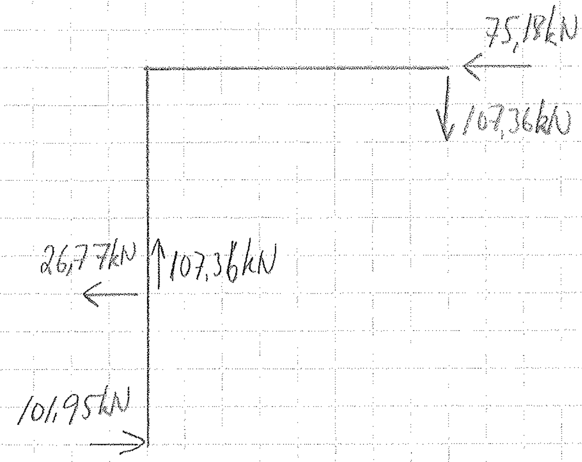
$107{,}36 \cdot 0{,}8 - 75{,}18 \cdot 0{,}6 - A_x \cdot 0{,}4 = 0$

$\Rightarrow \underline{\underline{A_x = 101{,}95\,kN}}$

$\sum F_x = 0$ gir $A_x + B_x = 75{,}18$

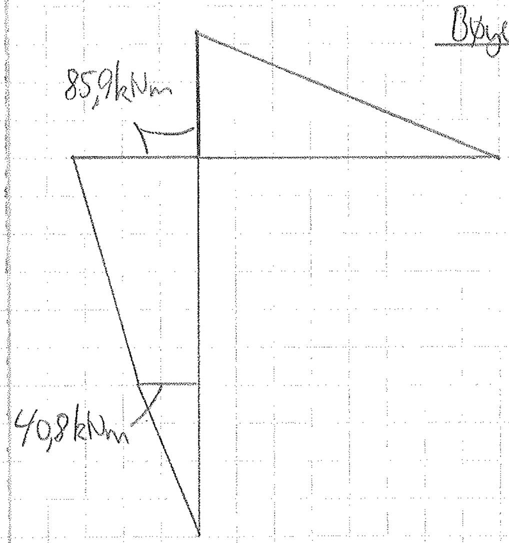
$\Rightarrow \underline{\underline{B_x = -26{,}77\,kN}}$

Belastningsdiagram:



c)

Bøyemomentdiagram



d)

$$\sigma_B = \frac{M}{W} \Rightarrow W_{min} = \frac{85,9 \cdot 10^6}{160} = 537 \cdot 10^3 \, mm^3 = 537 \, cm^3$$

HE200B har $W_x = 570 \, cm^3$ og $A = 78,1 \, cm^2$

Sjekker aksialspenningene i et snitt like under C:

$$\sigma_A = \frac{F}{A} = \frac{107,36 \cdot 10^3}{7810} = 13,7 \, MPa$$

$$\sigma_B = \frac{85,9 \cdot 10^6}{570 \cdot 10^3} = 150,7 \, MPa$$

$\sigma = \sigma_A + \sigma_B = 164,4 \, MPa$

Dvs ikke ok.

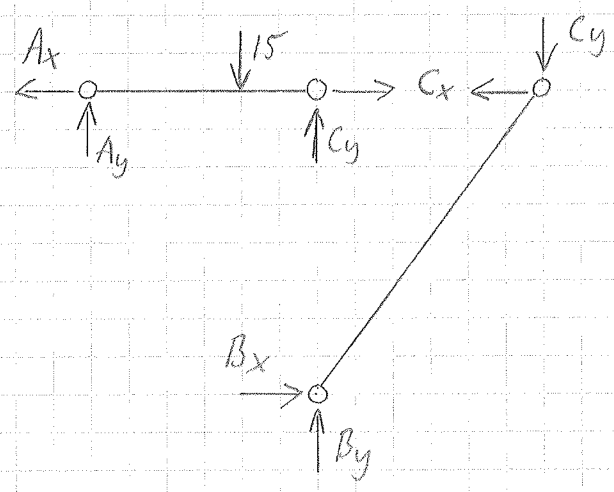
Går derfor opp en dimensjon og velger HE220B

Oppgave 2

a) I et ideelt fagverk skal alle belastninger angripe i knutepunktene. Dette er ikke tilfelle her. Derfor er konstruksjonen ikke et ideelt fagverk.

Ant. elementer : $e = 2$
Ant. ukjente : $r = 6$
} $r = 3e$ dvs statisk bestemt.

b) Fritt-legeme-diagram



Element AC:

$\Sigma M_A = 0$ gir $C_y \cdot 6 = 15 \cdot 4 \Rightarrow$ $C_y = 10 kN$

$\Sigma F_y = 0$ gir $A_y + C_y = 15 \Rightarrow$ $A_y = 5 kN$

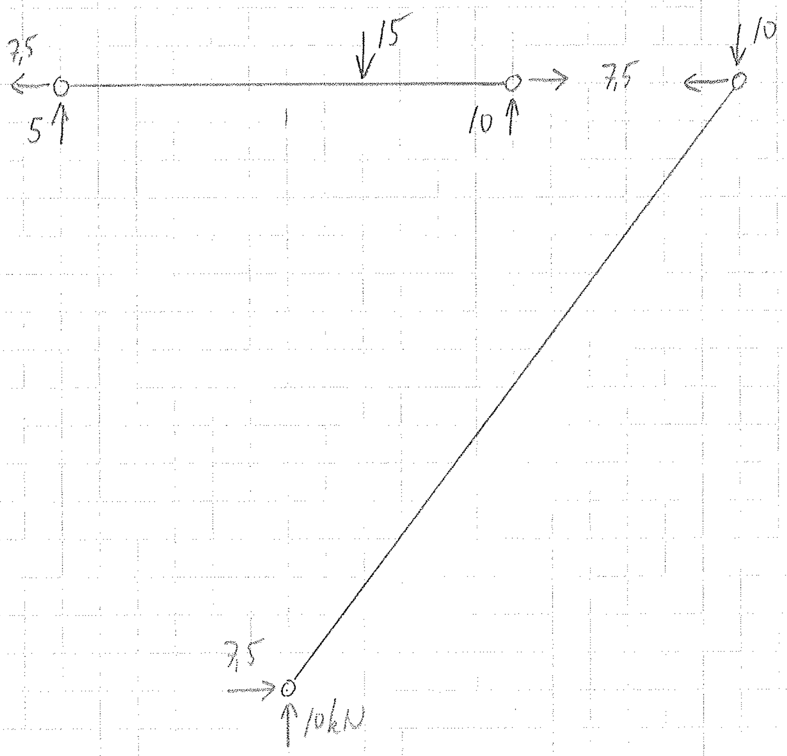
Hele konstr:

$\Sigma M_B = 0$ gir $A_x \cdot 8 = 15 \cdot 4 \Rightarrow$ $A_x = 7{,}5 kN$

$\Sigma F_y = 0$ gir $A_y + B_y = 15 \Rightarrow$ $B_y = 10 kN$

For øvrig ser vi lett at $A_x = B_x = C_x$

Belastningsdiagram



c) Største bøyemoment finner vi under punktlasten : $\underline{M} = 10 \cdot 2 = \underline{20 kNm}$

$$\sigma = \frac{M}{W} \quad \text{gir} \quad \underline{W_{min}} = \frac{20 \cdot 10^6}{160} = 125 \cdot 10^3 mm^3 = \underline{125 cm^3}$$

HE120B har $W_x = 144 cm^3$ og $A = 34,0 cm^2$

Sjekker de totale normalspenningene :

$$\left.\begin{aligned} \sigma_B &= \frac{20 \cdot 10^6}{144 \cdot 10^3} = 139 MPa \\ \sigma_A &= \frac{7500}{3400} = 2,2 MPa \end{aligned}\right\} \sigma_N = 141 MPa \text{ dvs ok}$$

Velger derfor $\underline{\underline{HE120B}}$

d) $l_k = \sqrt{8^2 + 6^2} = 10m$

HE120B har $I_0 = I_y = 318 cm^4$

$$F_k = \frac{\pi^2 E I_0}{l_k^2} = \frac{\pi^2 206000 \cdot 3180000}{10000^2} = 64,7kN$$

Aksiallast på element BC: $F_A = \sqrt{10^2 + 7,5^2} = 12,5kN$

Sikkerhetsfaktor i forhold til elastisk knekning: $\underline{n} = \frac{F_k}{F_A} = \frac{64,7}{12,5} = \underline{5,18}$

e) Hookes lov: $\sigma = E\varepsilon$, $\varepsilon = \frac{\Delta l}{l}$ og $\sigma = \frac{F}{A}$ gir oss

$$\Delta l = \frac{Fl}{EA}$$

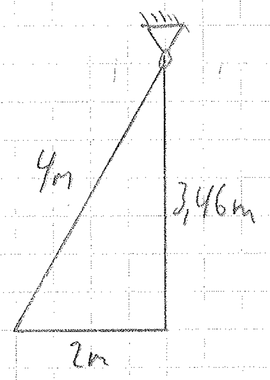
HE120B har $A = 34,0 cm^2$

$$\underline{\Delta l} = \frac{12500 \cdot 10000}{206000 \cdot 3400} = \underline{0,18mm}$$

Element BC blir altså 0,18mm kortere som følge av den ytre lasten

Oppgave 3

a)



Pytagoras: $2^2 + x^2 = 4^2$

$\Rightarrow x^2 = 16 - 4 = 12$

$\underline{x} = \sqrt{12} = \underline{3{,}46m}$

$G = 400 \cdot 9{,}81 = 3924N$

Momentlikevekt om opphengspunkt:

$F \cdot 3{,}46 = 3924 \cdot 2 \Rightarrow \underline{F = 2266N}$

b)

$M_v = 10 \cdot 5{,}5 = 55 \text{ tm}$

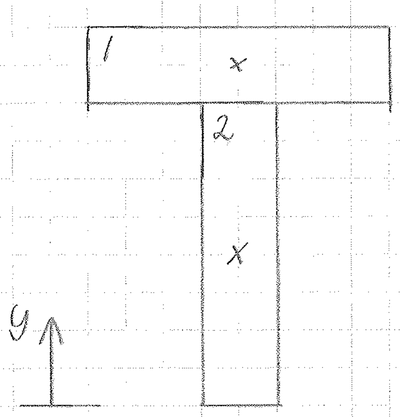
$M_s = 15 \cdot 2{,}25 + W \cdot 6{,}5$

Dimensjoneringskriterium $M_s = 1{,}6 M_v$

$\Rightarrow 33{,}75 + 6{,}5W = 1{,}6 \cdot 55$

$\underline{W = 8{,}35t}$

c)



$A_1 = 4000\,mm^2$

$y_{01} = 210\,mm$

$I_{01} = \frac{1}{12} 200 \cdot 20^3 = 133 \cdot 10^3\,mm^4$

$A_2 = 4000\,mm^2$

$y_{02} = 100\,mm$

$I_{02} = \frac{1}{12} 20 \cdot 200^3 = 13{,}3 \cdot 10^6\,mm^4$

$$\underline{\bar{Y}} = \frac{4000 \cdot 210 + 4000 \cdot 100}{8000} = \underline{155\,mm}$$

$$y_1 = y_{01} - \bar{y} = 210 - 155 = 55\,mm$$

$$y_2 = \bar{y} - y_{02} = 155 - 100 = 55\,mm$$

Steiners formel:

$$I_1 = I_{01} + y_1^2 A_1 = 133 \cdot 10^3 + 55^2 \cdot 4000 = 133 \cdot 10^3 + 12{,}1 \cdot 10^6 = 12{,}2 \cdot 10^6\,mm^4$$

$$I_2 = I_{02} + y_2^2 A_2 = 13{,}3 \cdot 10^6 + 12{,}1 \cdot 10^6 = 25{,}4 \cdot 10^6\,mm^4$$

$$\underline{I} = I_1 + I_2 = \underline{37{,}6 \cdot 10^6\,mm^4}$$

d)

$$\left.\begin{aligned} y_u &= 155\,mm \\ y_o &= 220 - 155 = 65\,mm \end{aligned}\right\}$$ dvs vi får størst spenninger i underkant

$$\underline{\sigma_{Bu}} = \frac{M}{I} y_u = \frac{20 \cdot 10^6}{37{,}6 \cdot 10^6} \cdot 155 = \underline{82{,}4\,MPa}$$