Mekanikk, ordinær eksamen 2/12 - 2009, Løsning

1)

a) FLD:

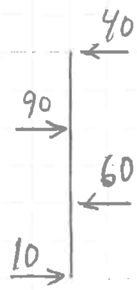
$$\sum \overset{\curvearrowright}{M}_B = 0 \Rightarrow 90\cdot 2 - 60\cdot 1 - A_x\cdot 3 = 0$$

$$\Rightarrow \underline{A_x = 40N}$$

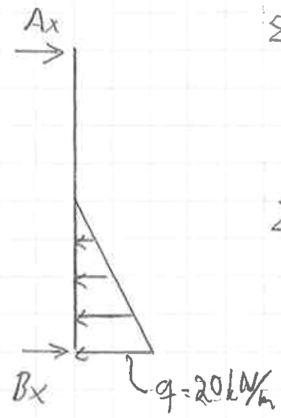
$$\sum \vec{F}_x = 0 \Rightarrow B_x + 90 - 60 - 40 = 0$$

$$\underline{B_x = 10N}$$

Belastningsdiagram



b) FLD:



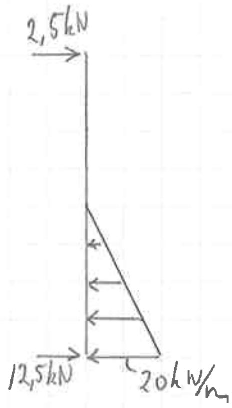
$$\sum \overset{\curvearrowright}{M}_B = 0 \Rightarrow A_x\cdot 3 - \tfrac{1}{2}20\cdot 1{,}5\cdot \tfrac{1}{3}1{,}5 = 0$$

$$\Rightarrow A_x = 2{,}5kN$$

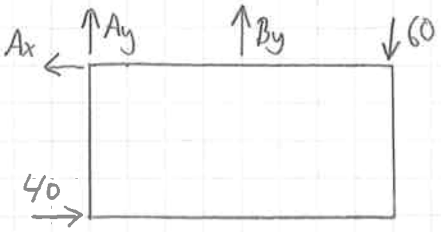
$$\sum \vec{F}_x = 0 \Rightarrow B_x + 2{,}5 - \tfrac{1}{2}20\cdot 1{,}5 = 0$$

$$\Rightarrow B_x = 12{,}5kN$$

Belastningsdiagram



c) FLD

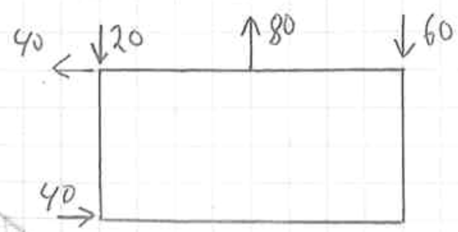


$$\sum \overset{\curvearrowright}{M}_A = 0 \Rightarrow 60\cdot 2{,}4 - 40\cdot 1{,}2 - B_y\cdot 1{,}2 = 0 \Rightarrow \underline{B_y = 80N}$$

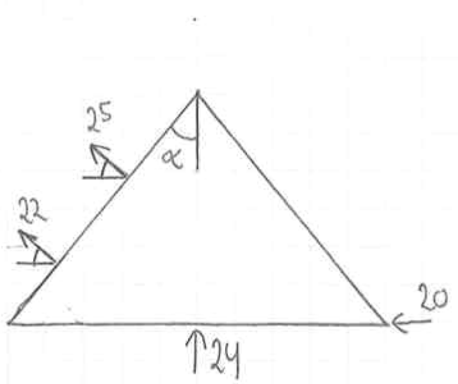
$$\sum \vec{F}_x = 0 \Rightarrow \underline{A_x = 40N}$$

$$\uparrow \sum F_y = 0 \Rightarrow A_y + 80 - 60 = 0 \Rightarrow \underline{A_y = -20N}$$

Belastningsdiagram



d)

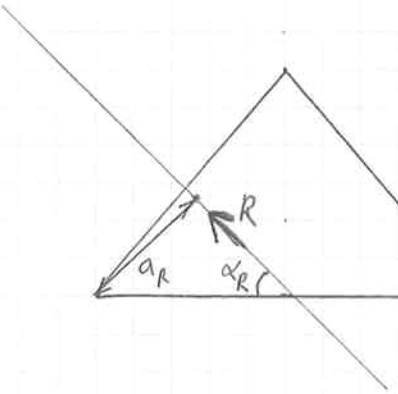


$$\tan\alpha = \frac{2{,}5}{3} \Rightarrow \alpha = 39{,}8^\circ$$

$$\overleftarrow{R_x} = 20 + 47 \cdot \cos 39{,}8 = 56{,}1$$

$$\uparrow R_y = 24 + 47 \sin 39{,}8 = 54{,}1$$

$$R = \sqrt{R_x^2 + R_y^2} = 77{,}9 \text{ kN}$$



$$\tan\alpha_R = \frac{54{,}1}{56{,}1} \Rightarrow \alpha_R = 44^\circ$$

$$a_R \cdot R = \Sigma M_P = 24 \cdot 2{,}5 + 22 \cdot 1{,}0 + 25 \cdot 2{,}5 = 144{,}5$$

$$\Rightarrow a_R = \frac{144{,}5}{77{,}9} = 1{,}85 \text{ m}$$

2a) FLD:

Hele konstr

$$\Sigma \overset{\curvearrowright}{M_A} = 0 \Rightarrow 20 \cdot 1{,}5 - C_x \cdot 5 = 0 \Rightarrow \underline{C_x = 6 \text{ kN}}$$

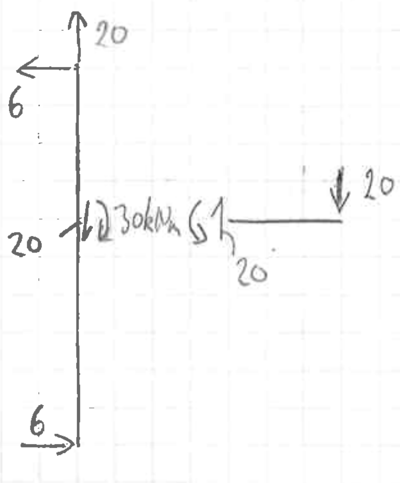
$$\Sigma \vec{F_x} = 0 \Rightarrow \underline{A_x = 6 \text{ kN}}$$

Element BD:

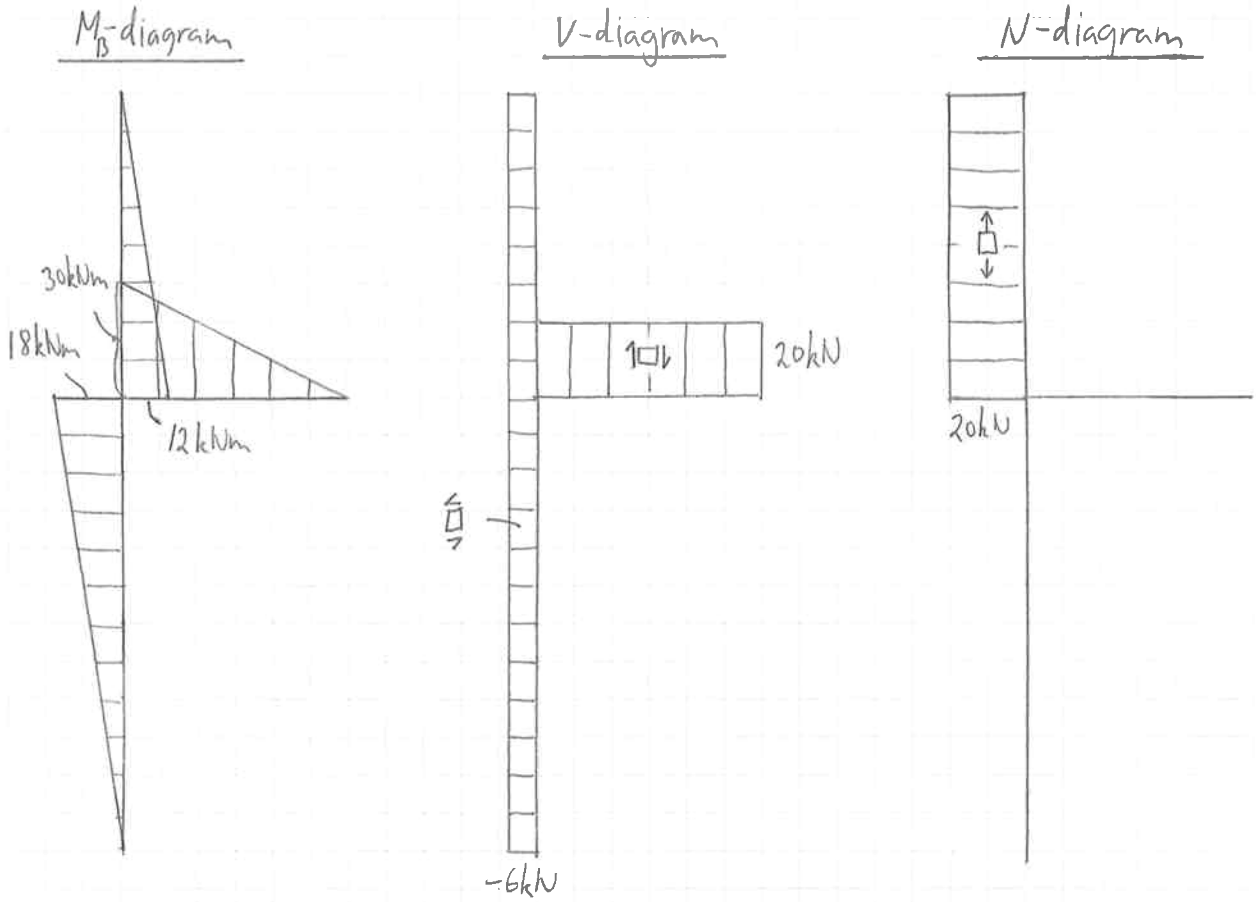
$$\Sigma \overset{\curvearrowright}{M_B} = 0 \Rightarrow 20 \cdot 1{,}5 - M_B = 0 \Rightarrow \underline{M_B = 30 \text{ kNm}}$$

$$\uparrow \Sigma F_y = 0 \Rightarrow \underline{B_y = 20 \text{ kN}}$$

Belastnings-diagram:



b)



c)

$$\sigma_{till} = \frac{\sigma_F}{n} = \frac{235}{1{,}2} = 195{,}8\,MPa$$

$$\sigma_{B,max} = \sigma_{till} \Rightarrow \frac{M_{dim}}{W_{krav}} = \sigma_{till}$$

$$W_{krav} = \frac{M_{dim}}{\sigma_{till}} = \frac{30 \cdot 10^6}{195{,}8} = 153191\,mm^3 = 153\,cm^3$$

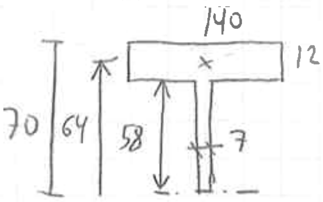
HE140B, har $W_x = 216\,cm^3$ og kan derfor brukes

d) I arealsenteret har vi kun skjærspenninger

HE140B: $I_x = 1510 \cdot 10^4\,mm^4$

$b = 7\,mm$ (stegtykkelse)

Arealmoment av halve HE140B:



$$S = 140 \cdot 12 \cdot 64 + 58 \cdot 7 \cdot 29 = 119294$$

Bøyindusert skjærspenning:

$$\tau = \frac{V}{Ib} S = \frac{20000}{1510 \cdot 10^4 \cdot 7} \cdot 119294 = 22{,}6 \text{ MPa}$$

Gjør om skjærsp. til jevnføringsspenning:

$$\sigma_j = \sqrt{3}\,\tau = \underline{39{,}1 \text{ MPa}}$$

Alternativ løsning (gir 70% rett)

$$A_s = (140 - 2 \cdot 12) \cdot 7 = 812 \text{ mm}^2$$

$$\tau = \frac{V}{A_s} = \frac{20000}{812} = 24{,}6 \text{ MPa}$$

$$\sigma_j = \sqrt{3}\,\tau = \underline{42{,}7 \text{ MPa}}$$

3a) Statisk bestemthet:

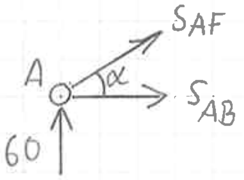
Ant. staver: $s = 9$
Ant. opplagerkrefter: $o = 3$
Ant. knutepunkt: $k = 6$

$\Big\}$ $s + o = 2k$ dvs statisk bestemt!

Konstr. er symmetrisk både mhp geometri og belastning. Halve lasten går dermed i venstre opplagring og $A_y = E_y = 60\,kN$

$$\tan\alpha = \frac{2,5}{5,0} \Rightarrow \alpha = 26,6^\circ$$

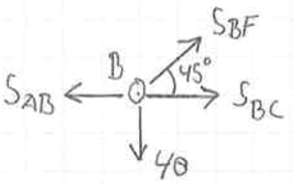
Knutepunkt A:



$$\uparrow \Sigma F_y = 0 \Rightarrow 60 + S_{AF} \cdot \sin 26,6^\circ = 0 \Rightarrow \underline{S_{AF} = -134\,kN}$$

$$\Sigma \vec{F}_x = 0 \Rightarrow S_{AB} + (-134)\cos 26,6^\circ = 0 \Rightarrow \underline{S_{AB} = 120\,kN}$$

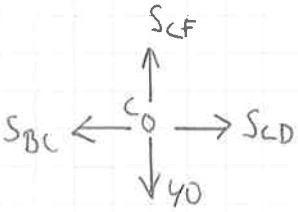
b) Knutepunkt B:



$$\uparrow \Sigma F_y = 0 \Rightarrow S_{BF} \sin 45^\circ - 40 = 0 \Rightarrow \underline{S_{BF} = 56,6\,kN}$$

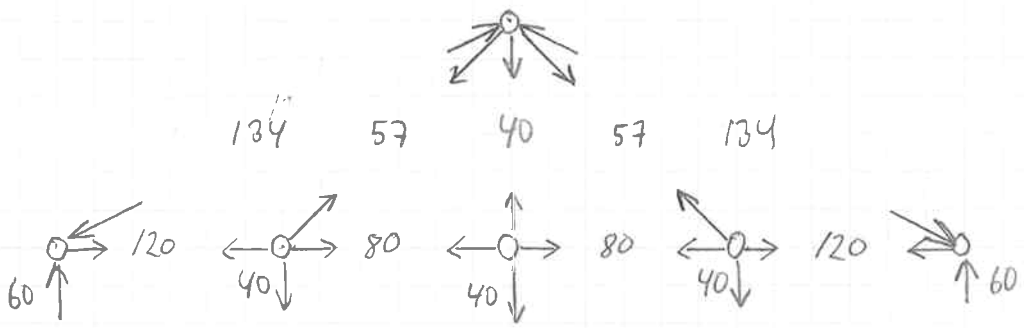
$$\Sigma \vec{F}_x = 0 \Rightarrow S_{BC} + 56,6 \cdot \cos 45^\circ - 120 = 0 \Rightarrow \underline{S_{BC} = 80\,kN}$$

Knutepunkt C:



$$\uparrow \Sigma F_y = 0 \Rightarrow S_{CF} = 40\,kN$$

Belastningsdiagram



c)

$\varnothing 120 \times 6$: $D = 120$ mm, $d = 108$ mm

$$A = \frac{\pi}{4}(D^2 - d^2) = 2149 \text{ mm}^2$$

$$\sigma_A = \frac{F}{A} = \frac{134000}{2149} = \underline{62{,}4 \text{ MPa}}$$

$$n = \frac{\sigma_F}{\sigma_A} = \frac{235}{62{,}4} = \underline{3{,}8}$$

Lengde på stav AF: $L = \sqrt{2500^2 + 5000^2} = 5590$ mm

Forkortelse av stav AF:

$$\underline{\Delta L} = \frac{FL}{EA} = \frac{134000 \cdot 5590}{206000 \cdot 2149} = \underline{1{,}7 \text{ mm}}$$

d) $I_0 = \frac{\pi}{64}\left(D^4 - d^4\right) = 3,5 \cdot 10^6\ mm^4$

$$i = \sqrt{\frac{I_0}{A}} = \sqrt{\frac{3,5 \cdot 10^6}{2149}} = 40,36\ mm$$

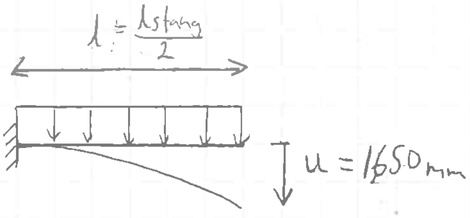
$$\lambda = \frac{l_k}{i} = \frac{5590}{40,36} = 138,5 > 105$$ dvs elastisk knekking

$$F_k = \frac{\pi^2 E I_0}{l_k^2} = \frac{\pi^2\, 206000 \cdot 3,5 \cdot 10^6}{5590^2} = 228\,kN$$

$$\underline{n_k} = \frac{F_k}{S_{AF}} = \frac{228}{134} = \underline{1,7}$$

4a) Stangen kan betraktes som to stk utkragerbjelker med jevnt fordelt last.

$$q = \rho g A = 7850\,\frac{kg}{m^3} \cdot 9,81\,\frac{N}{kg} \cdot \frac{\pi}{4} \cdot 0,005^2\,m^2 = 1,512\ N/m = 1,512 \cdot 10^{-3}\,\frac{N}{mm}$$



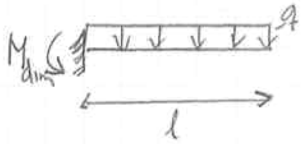
$$I = \frac{\pi}{64}\, s^4 = 30,68\ mm^4$$

$$u_{max} = \frac{qL^4}{8EI}$$

$$\Rightarrow L = \sqrt[4]{\frac{8EIu}{q}} = \sqrt[4]{\frac{8 \cdot 206000 \cdot 30,68 \cdot 1650}{1,512 \cdot 10^{-3}}} = 2725\ mm$$

Stangen berører bakken hvis $L_{stang} = 2 \cdot 2725 = \underline{\underline{5450\,mm}}$

b) Bruker samme modell som i (a) og setter opp et uttrykk for det største bøyemoment som å finner ved arbeiderens skulder:



$$M_{dim} = q \cdot l \cdot \frac{l}{2} = \frac{1}{2} q l^2$$

Stangstålets motstandsmoment $W = \frac{I_x}{\frac{d}{2}} = \frac{30,68}{2,5} = 12,27 \text{ mm}^3$

Flyting skjer hvis

$$\sigma_{B,max} = \frac{M_{dim}}{W} = \sigma_F$$

$$\Rightarrow \frac{qL^2}{2W} = \sigma_F \Rightarrow L = \sqrt{\frac{2W\sigma_F}{q}} = \sqrt{\frac{2 \cdot 12,27 \cdot 235}{1,512 \cdot 10^{-3}}} = 1953 \text{ mm}$$

Stangen begynner å flyte hvis $L_{stang} = 2 \cdot 1953 = \underline{\underline{3906 \text{ mm}}}$