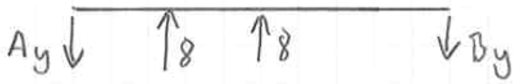
Mekanikkeksamen 24.02.2010 – Løsning

Oppg 1

a) FLD:



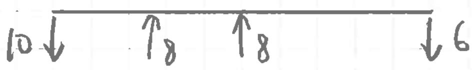
$\curvearrowleft \Sigma M_A = 0 \Rightarrow B_y \cdot 5 - 8 \cdot 1{,}25 - 8 \cdot 2{,}5 = 0$

$\Rightarrow \underline{B_y = 6\,kN}$

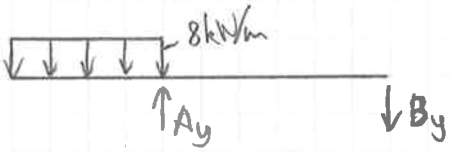
$\uparrow \Sigma F_y = 0 \Rightarrow 8 + 8 - A_y - 6 = 0$

$\Rightarrow \underline{A_y = 10\,kN}$

BD:



b) FLD:



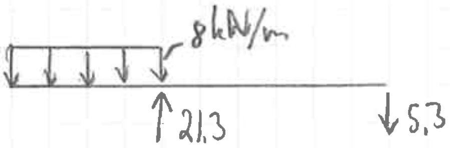
$\curvearrowleft \Sigma M_A = 0 \Rightarrow B_y \cdot 3 - 8 \cdot 2 \cdot 1 = 0$

$\Rightarrow \underline{B_y = 5{,}3\,kN}$

$\uparrow \Sigma F_y = 0 \Rightarrow A_y - 5{,}3 - 8 \cdot 2 = 0$

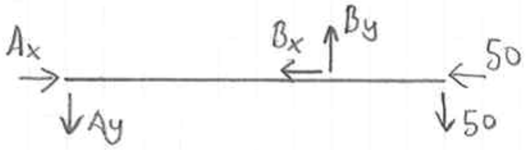
$\Rightarrow \underline{A_y = 21{,}3\,kN}$

BD:



Oppgave 2

a) FLD:



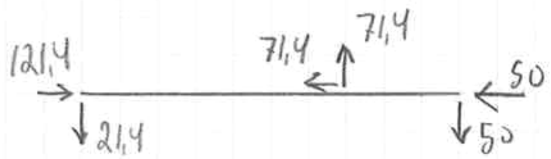
$$\sum \overset{\curvearrowleft}{M_A} = 0 \Rightarrow 50 \cdot 5 - B_y \cdot 3{,}5 = 0 \Rightarrow \underline{B_y = 71{,}4\,kN}$$

kraften i B har samme retning som staget (45°) og vi får $\underline{B_x = B_y}$

$$\uparrow \sum F_y = 0 \Rightarrow 71{,}4 - 50 - A_y = 0 \Rightarrow \underline{A_y = 21{,}4\,kN}$$

$$\overset{\rightarrow}{\sum} F_x = 0 \Rightarrow A_x - 71{,}4 - 50 = 0 \Rightarrow \underline{A_x = 121{,}4\,kN}$$

BD:



b)

$$A_{stag} = \frac{\pi}{4} 20^2 = 314\,mm^2$$

Strekkraft i stag: $B = \sqrt{B_x^2 + B_y^2} = \sqrt{2}\, B_x = \underline{101\,kN}$

$$\underline{\underline{\sigma_{stag}}} = \frac{F}{A} = \frac{101000}{314} = \underline{\underline{321{,}6\,MPa}}$$

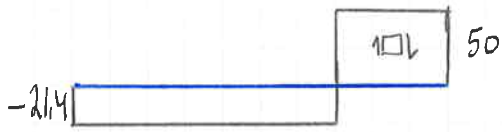
$$L_{stag} = \sqrt{2} \cdot 3{,}5 = 4950\,mm$$

$$\underline{\underline{\Delta L}} = \frac{FL}{EA} = \frac{101000 \cdot 4950}{206000 \cdot 314} = \underline{\underline{7{,}7\,mm}}$$

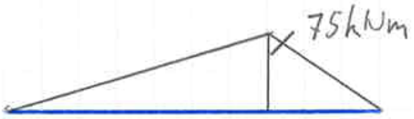
c)

N-diagram:

V-diagram:



M-diagram:



d)

IPE270: $W_x = 429\,cm^3$

$A = 4590\,mm^2$

$$\sigma_{Bo} = \frac{M_{dim}}{W_x} = \frac{75 \cdot 10^6}{429 \cdot 10^3} = 174{,}8\,MPa$$

$$\sigma_A = \frac{N}{A} = \frac{121400}{4590} = 26{,}4\,MPa$$

$$\underline{\underline{\sigma_{MAX}}} = \sigma_{Bo} + \sigma_A = \underline{\underline{201{,}2\,MPa}}$$

$$\underline{\underline{n_F}} = \frac{235}{201{,}2} = \underline{\underline{1{,}17}}$$

e)

$$I_{red} = 5790 - \frac{1}{12} \cdot 0{,}66 \cdot 5{,}0^3 = 5783\ \text{cm}^4$$

$$A_{Red} = 4590 - 6{,}6 \cdot 50 = 4260\ \text{mm}^2$$

$$\sigma_{BO} = \frac{M}{I} y = \frac{75 \cdot 10^6}{57{,}83 \cdot 10^6} \cdot \frac{270}{2} = 175{,}1\ \text{MPa}$$

$$\sigma_A = \frac{N}{A} = \frac{121400}{4260} = 28{,}5\ \text{MPa}$$

$$\underline{\underline{\sigma_{max}}} = \sigma_{BO} + \sigma_A = \underline{\underline{203{,}6\ \text{MPa}}}$$

f)

$$A_{steg} = (270 - 50 - 2 \cdot 10{,}2) \cdot 6{,}6 = 1317\ \text{mm}^2$$

$$\tau = \frac{V}{A_s} = \frac{50 \cdot 10^3}{1317} = 38{,}0\ \text{MPa}$$

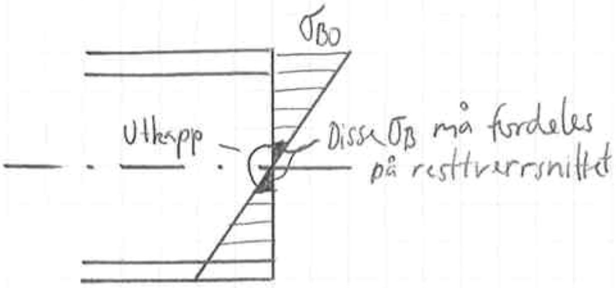
Beregner σ_B i steget rett under flensen

$$\sigma_B = \frac{M}{I_{Red}} y = \frac{75 \cdot 10^6}{57{,}83 \cdot 10^6} \cdot (135 - 10{,}2) = 161{,}9\ \text{MPa}$$

$$\sigma = \sigma_B + \sigma_A = 161{,}9 + 28{,}5 = 190{,}4\ \text{MPa}$$

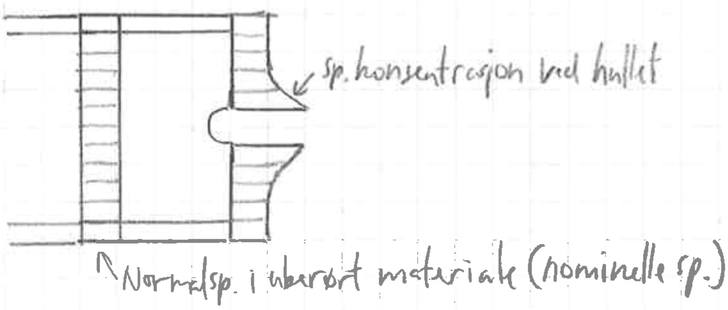
$$\underline{\sigma_j} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{190{,}4^2 + 3 \cdot 38{,}0^2} = \underline{201{,}4\ \text{MPa}}$$

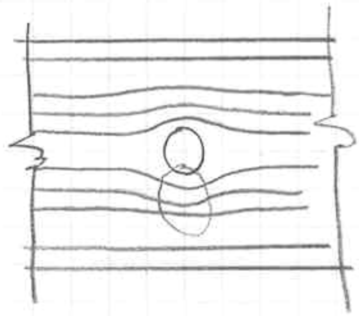
g) Hullet er plassert ved tverrsnittets nøytralakse hvor vi har små bøyespenninger og dermed blir det minimalt med bøyesp. som må fordeles på resttverrsnittet. Hvis utkappet ble gjort i flensen så ville spenningsbildet bli vesentlig forverret



Utkappet vil ha en større innvirkning på aksialsp. og skjærsp. Men som vi ser er σ_A og τ relativt små og av underordnet betydning i forhold til σ_B

Ved brå overganger i et tverrsnitts geometri (f.eks. ved utkapp) kan vi lokalt få svært store spenninger, og dette kalles spenningskonsentrasjoner. For å unngå dette bør vi unngå skarpe hjørner, plutselige dimensjonsendringer osv.





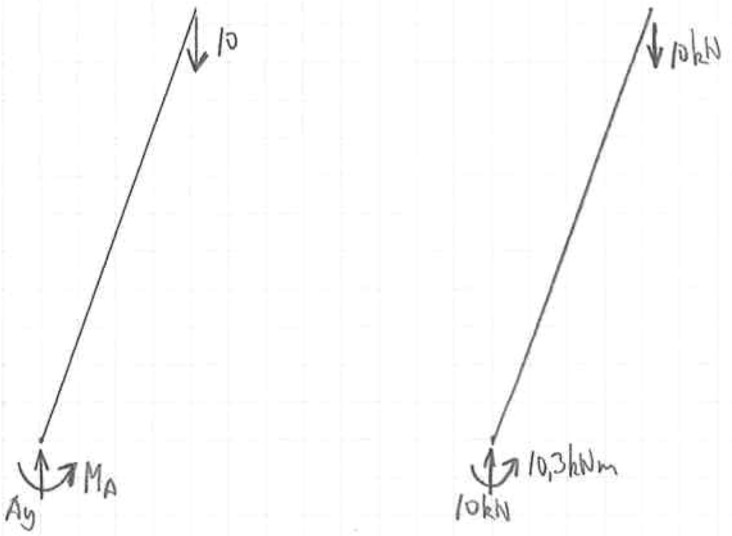
b)

$$F_k = \frac{\pi^2 E I_o}{l_k^2} = \frac{\pi^2 \cdot 206000 \cdot 4200000}{3500^2} = 697\,kN$$

$$\underline{\underline{n_k}} = \frac{697}{121{,}4} = \underline{\underline{5{,}7}}$$

Oppgave 3

a) FLD: BD:



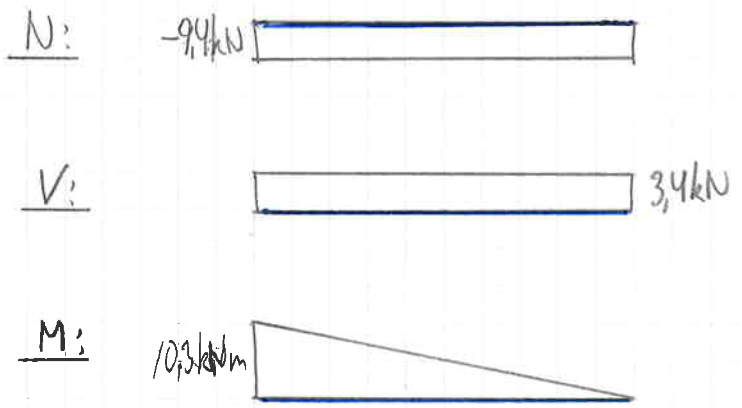
$$\uparrow \sum F_y = 0 \Rightarrow A_y = 10kN$$

$$\sum \overset{\curvearrowleft}{M_A} = 0 \Rightarrow 10 \cdot 3 \cdot \sin 20 - M_A = 0 \Rightarrow M_A = 10{,}3kNm$$

b) Her er det en fordel å tegne et nytt BD hvor kreftene er dekomponert i bjelkeaksen og på tvers av bjelken. Legger bjelken ned

$A_x = B_x = 10 \cos 20 = 9{,}4$

$A_y = B_y = 10 \sin 20 = 3{,}4$



c)

$$\sigma_{tillatt} = \frac{\sigma_F}{n} = \frac{235}{1{,}5} = 156{,}7 \text{ MPa}$$

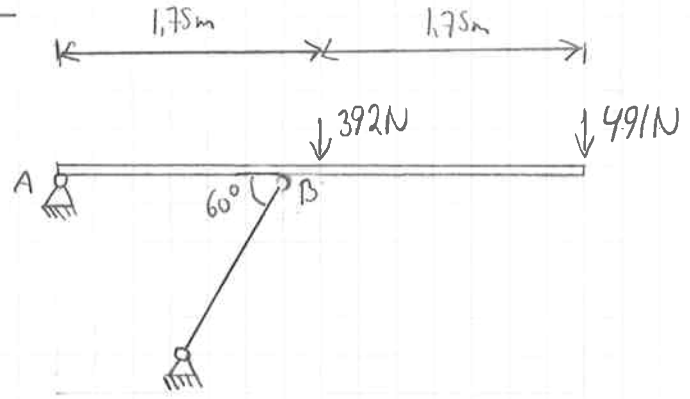
$$W_{tillatt} = \frac{M_{dim}}{\sigma_{tillatt}} = \frac{10{,}3 \cdot 10^6}{156{,}7} = 65745 \text{ mm}^3 = 65{,}7 \text{ cm}^3$$

IPE140 har $W_x = 77{,}3 \text{ cm}^3$ og $A = 1640 \text{ mm}^2$

$$\left.\begin{aligned} \sigma_B &= \frac{10{,}3 \cdot 10^6}{77{,}3 \cdot 10^3} = 133{,}2 \text{ MPa} \\ \sigma_A &= \frac{9400}{1640} = 5{,}7 \end{aligned}\right\} \sigma = \sigma_B + \sigma_A = 138{,}9 \text{ MPa} < \sigma_{tillatt} \text{ dvs ok}$$

Oppgave 4

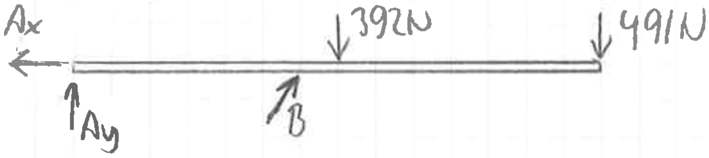
a) Modell:



$F_{person} = 9.81 \cdot 50 = 491\,N$

$F_{planke} = 9,81 \cdot 40 = 392\,N$

FLD:



$$\overset{\curvearrowleft}{\sum M_A} = 0 \Rightarrow 392 \cdot 1,75 + 491 \cdot 3,5 - B \cdot \sin 60 \cdot 1,5 = 0$$

$$\Rightarrow \underline{\underline{B = 1851\,N}}$$

b)

$$\uparrow \sum F_y = 0 \Rightarrow A_y + 1851 \sin 60 - 392 - 491 = 0 \Rightarrow \underline{A_y = -720\,N}$$

$$\overrightarrow{\sum F_x} = 0 \Rightarrow B \cos 60 - A_x = 0 \Rightarrow \underline{A_x = 925\,N}$$

Kraft som virker i A:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{925^2 + 720^2} = \underline{1172\,N}$$

Kraftens retning i A:

$$\tan \varphi = \frac{A_y}{A_x} = \frac{720}{925} = 0,78 \Rightarrow \underline{\varphi = 37,9^\circ}$$

Det er motkraften som virker på bolten: