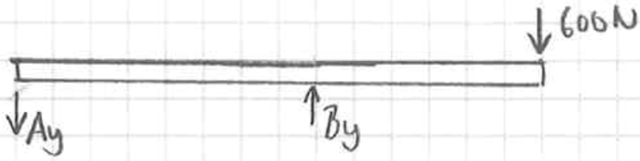
Eksamen Mekanikk 2011B, 22/2-2012 Løsning

Oppg 1

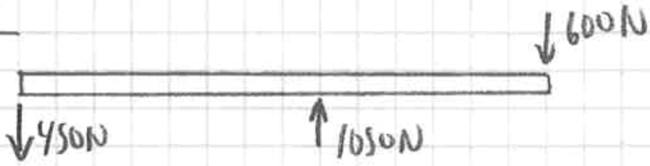
a) FLD:



$$\curvearrowright \sum M_B = 0 \Rightarrow 600 \cdot 1{,}5 - A_y \cdot 2 = 0 \quad \Rightarrow A_y = 450N$$

$$\uparrow \sum F_y = 0 \Rightarrow B_y - 600 - 450 = 0 \quad \Rightarrow B_y = 1050N$$

BD

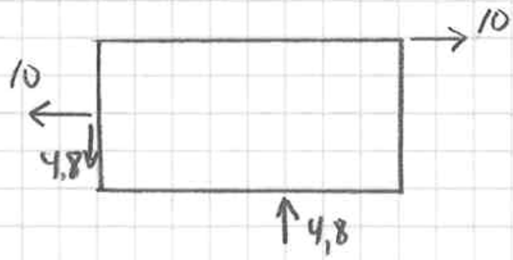


b) FLD

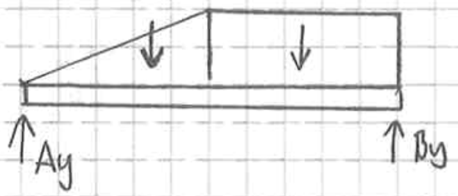
$$\curvearrowright \sum M_A = 0 \Rightarrow 10 \cdot 1{,}2 - B_y \cdot 2{,}5 = 0 \Rightarrow B_y = 4{,}8kN$$

Ser umiddelbart at $A_x = 10$ og $A_y = 4{,}8$

BD:



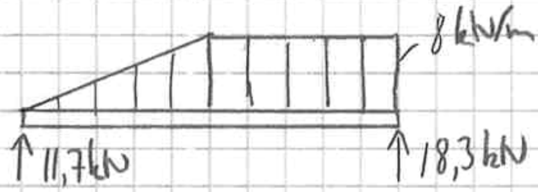
c) FLD



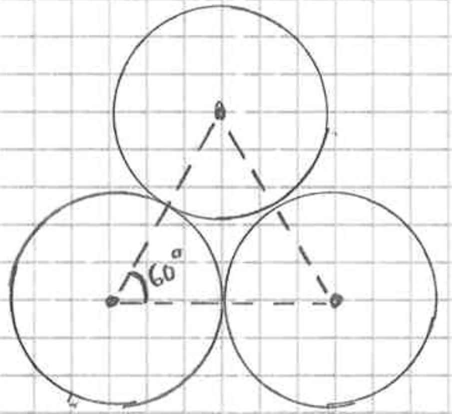
$$\curvearrowleft \Sigma M_A = 0 \Rightarrow \underbrace{\tfrac{1}{2}\,2{,}5\cdot 8}_{F}\cdot\underbrace{\tfrac{2}{3}\cdot 2{,}5}_{a} + 2{,}5\cdot 8\cdot 3{,}75 - B_y\cdot 5 = 0 \quad \Rightarrow B_y = 18{,}3\text{ kN}$$

$$\Sigma F_y = 0 \Rightarrow A_y + 18{,}3 - \tfrac{1}{2}\,2{,}5\cdot 8 - 2{,}5\cdot 8 = 0 \quad \Rightarrow A_y = 11{,}7\text{ kN}$$

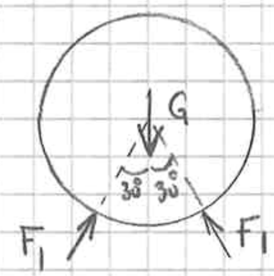
BD:



d)



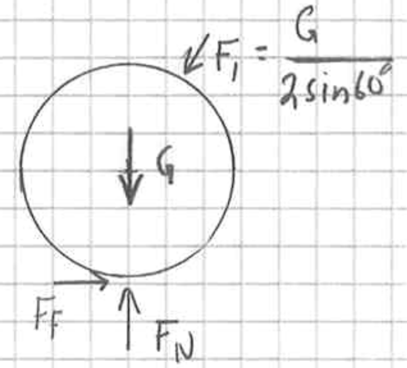
FLD av øverste bolt



Kontaktkraft F_1 virker normalt på overflaten $\quad \uparrow \Sigma F_y = 0 \Rightarrow 2\cdot F_1\cdot \sin 60° - G = 0$

FLD av nedre bolter

$$F_1 = \frac{G}{2\sin 60°} =$$



$$F_N = G + G/2 = \tfrac{3}{2}G$$

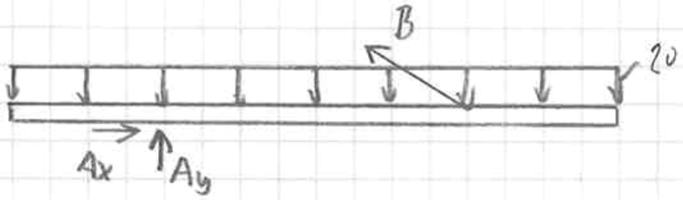
$$F_F = F_1\cos 60° = \frac{G\cos 60}{2\sin 60} = \frac{G}{2\tan 60°}$$

Nedre bolt glir hvis $F_F = \mu F_N$ dvs $\quad \underline{\underline{\mu_{min}}} = \frac{G}{2\tan 60°}\cdot\frac{2}{3G} = \frac{1}{3\tan 60°} = \frac{\sqrt{3}}{9} = \underline{\underline{0{,}19}}$

Oppgave 2

a) Tauets retning : $\alpha = \tan^{-1}\left(\frac{3}{6}\right) = 26{,}6^\circ$

FLD:



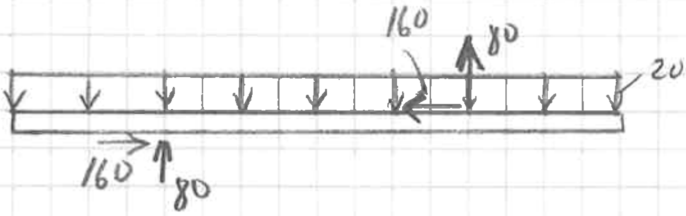
$$\curvearrowleft \sum M_B = 0 \Rightarrow A_y \cdot 4 - 20 \cdot 8 \cdot 2 = 0 \quad \Rightarrow A_y = 80\,kN$$

$$\uparrow \sum F_y = 0 \Rightarrow 80 + B_y - 20 \cdot 8 = 0 \quad \Rightarrow B_y = 80\,kN$$

Trigonometri : $\frac{B_x}{B_y} = \frac{6}{3} \Rightarrow B_x = 160\,kN$

$$\sum F_x = 0 \Rightarrow A_x = 160\,kN$$

BD



b)

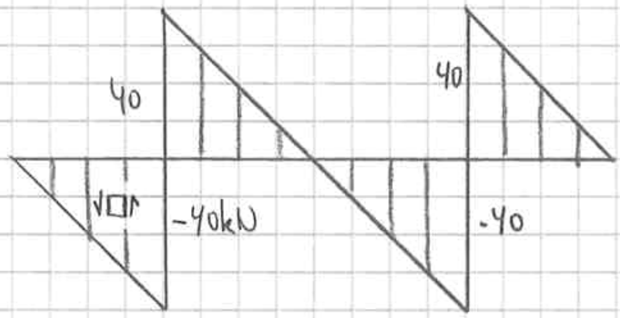
M

$M_A = 20\cdot 2\cdot 1 = 40\text{kNm}$

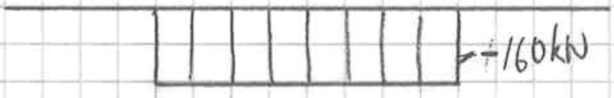
Symmetri tilsier at vi bare behøver å regne M for midtpkt:

$M_c = 20\cdot 4\cdot 2 - 80\cdot 2 = 0$

V



N



c) $M_{dim} = 40\text{kNm}$

$$\sigma_{till} = \frac{R_e}{1{,}4} = 253\text{MPa}$$

$$W_{krav} = \frac{M_{dim}}{\sigma_{till}} = \frac{40\cdot 10^6}{253} = 158\text{cm}^3$$

Prøver IPE 200 med $W_x = 194\text{cm}^3$ og $A = 28{,}5\text{cm}^2$

$$\sigma_B = \frac{40\cdot 10^6}{194\cdot 10^3} = 206\text{MPa} \qquad \sigma_A = \frac{N}{A} = \frac{160000}{2850} = 56\text{MPa}$$

$\sigma_A + \sigma_B = 262\text{MPa} > \sigma_{till}$ dvs ikke ok

IPE 220: $W_x = 252\text{cm}^3$, $A = 33{,}4\text{cm}^2 \Rightarrow \sigma_A + \sigma_B = \frac{40000}{252} + \frac{160000}{3340} = 206\text{MPa} < \sigma_{till}$

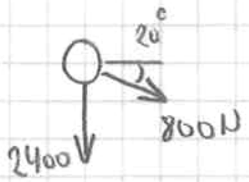
dvs ok

Oppgave 3

a) Løftekraft $F = \frac{1}{3} \cdot 2400 = 800N$

Resultantkraft i C : $\rightarrow R_x = 800 \cos 20^\circ = 752N$

$\downarrow R_y = 2400 + 800 \sin 20^\circ = 2674 N$



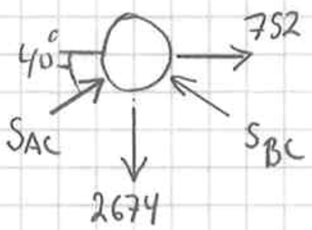
$$R = \sqrt{R_x^2 + R_y^2} = 2777 N$$

$$\varphi_R = \tan^{-1}\left(\frac{R_x}{R_y}\right) = 16^\circ$$

b)

$$\varphi_{AC} = \tan^{-1}\left(\frac{2,5}{3,0}\right) = 40^\circ$$

Ser på knutepunkt C :



$\uparrow \sum F_y = 0 \Rightarrow S_{AC} \sin 40^\circ + S_{BC} \sin 40^\circ - 2674 = 0$

$\Rightarrow S_{AC} + S_{BC} = 4160$ (1)

$\rightarrow \sum F_x = 0 \Rightarrow S_{AC} \cos 40^\circ - S_{BC} \cos 40^\circ + 752 = 0$

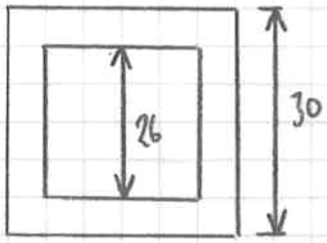
$\Rightarrow S_{AC} = S_{BC} - 982$ (2)

Setter (2) inn i (1):

$S_{BC} - 982 + S_{BC} = 4160 \Rightarrow \underline{S_{BC} = 2571N}$

(2): $\underline{S_{AC}} = 2571 - 982 = \underline{1589N}$

c)



$$A = 30^2 - 26^2 = 224 \text{ mm}^2$$

$$I_x = I_y = \frac{1}{12}\left(30^4 - 26^4\right) = 29419 \text{ mm}^4$$

$$\sigma_A = \frac{F}{A} = \frac{2571}{224} = \underline{11{,}5 \text{ MPa}}$$

$$i = \sqrt{\frac{I}{A}} = \sqrt{\frac{29419}{224}} = 11{,}5 \text{ mm}$$

$$L_k = L = \sqrt{2500^2 + 3000^2} = 3905 \text{ mm}$$

$$\lambda = \frac{L_k}{i} = \frac{3905}{11{,}5} = \underline{341}$$

$$F_k = \frac{\pi^2 E I}{L_k^2} = \frac{\pi^2 \cdot 70000 \cdot 29419}{3905^2} = 1333 \text{ N}$$

$S_{BC} > F_k$ dvs vi får elastisk knäckning !

Oppgave 4

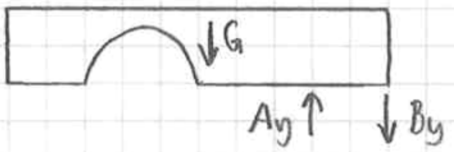
a) $$\bar{X}_{redusert} = \frac{A_{intakt} \cdot x_{intakt} - A_{utkapp} \cdot x_{utkapp}}{A_{redusert}}$$

$$\bar{X}_{red} = \frac{60 \cdot 200 \cdot 60 - \frac{1}{2}\pi \cdot 30^2 \cdot 90}{60 \cdot 200 - \frac{1}{2}\pi \cdot 30^2} = \frac{592765}{10586} = 56\,cm$$

$$\rho_{stål} = 7850\,kg/m^3 = 7850 \cdot 10^{-6}\,kg/cm^3$$

$$G = \rho g A t = 7850 \cdot 10^{-6} \cdot 9{,}81 \cdot 10586 \cdot 10 = 8152\,N$$

b)



$$\curvearrowleft \Sigma M_A = 0 \Rightarrow B_y \cdot 40 - 8152 \cdot 56 = 0 \Rightarrow B_y = 11413\,N$$

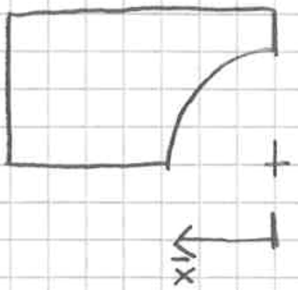
$$\uparrow \Sigma F_y = 0 \Rightarrow A_y - 11413 - 8152 = 0 \Rightarrow A_y = 19565\,N$$

Største bøyemoment finner vi i snittet ved A:

$$M_A = 8152 \cdot 560 = 4{,}57 \cdot 10^6\,Nmm$$

$$\sigma = \frac{M}{W} = \frac{4{,}57 \cdot 10^6}{\frac{1}{6} \cdot 100 \cdot 400^2} = 1{,}7\,MPa$$

c) Må regne tyngepunkt av nytt areal:



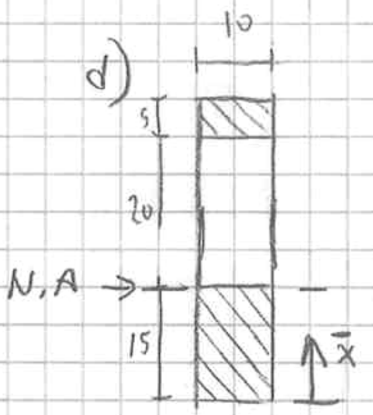
$$\bar{x} = \frac{70 \cdot 40 \cdot 35 - \underbrace{\frac{1}{4}\pi \cdot 30^2 \cdot \frac{4}{3\pi}}_{\frac{30^3}{3}} \cdot 30}{70 \cdot 40 - \frac{1}{4}\pi \cdot 30^2} = \frac{89000}{2093} = 42{,}5\ \text{cm}$$

$$G = 7850 \cdot 10^{-6} \cdot 9{,}81 \cdot 2093 \cdot 10 = 1611{,}8\ \text{N}$$

$$M = 1611{,}8 \cdot 425 = 6{,}85 \cdot 10^5$$

$$\sigma = \frac{M}{W} = \frac{6{,}85 \cdot 10^5}{\frac{1}{6} \cdot 100 \cdot 100^2} = 4{,}1\ \text{MPa}$$

d)



Bestemmer først N.A-beliggenhet

$$\bar{x} = \frac{15 \cdot 10 \cdot 7{,}5 + 5 \cdot 10 \cdot 37{,}5}{200} = 15\ \text{cm}$$

$$I = \frac{1}{12} 10 \cdot 5^3 + 5 \cdot 10 \cdot 22{,}5^2 + \frac{1}{12} 10 \cdot 15^3 + 15 \cdot 10 \cdot 7{,}5^2$$

$$I = 36667\ \text{cm}^4$$

$$\sigma_B = \frac{M}{I} y = \frac{100000 \cdot 1500}{36667 \cdot 10^4} \cdot 250 = \underline{102\ \text{MPa}}$$