

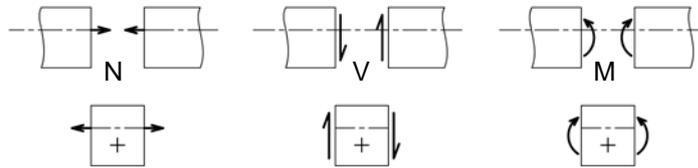


**Snittkrefter**

Sammenheng mellom lastintensitet, skjærkraft og bøyemoment

$$\frac{dV}{dx} = -q, \quad \frac{dM}{dx} = V$$

Fortegnsregler:



**Materialegenskaper**

	<u>Stål</u>	<u>Aluminium</u>
Egenvekt, $\rho$	7,85t/m <sup>3</sup>	2,7t/m <sup>3</sup>
E-modul, E	206 000MPa	70 000MPa
Temperaturutvidelseskoeff., $\alpha$	11·10 <sup>-6</sup> m/m°C	23,8·10 <sup>-6</sup> m/m°C

**Fasthetslære, grunnleggende**

Aksialspenninger:  $\sigma_A = \frac{F}{A}$

Tøyning:  $\varepsilon = \frac{\Delta L}{L}$

Termisk tøyning:  $\varepsilon_T = \alpha \cdot \Delta T$

Hookes lov:  $\sigma = E\varepsilon$

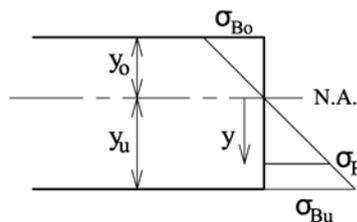
Forlengelse av aksialbelastet stav:  $\Delta L = \frac{FL}{EA}$

**Bøyespenningsformelen**

$$\sigma_B = \frac{M}{I_x} y$$

$$\sigma_{Bo} = \frac{M}{W_{xo}} \text{ hvor } W_{xo} = \frac{I_x}{y_o}$$

$$\sigma_{Bu} = \frac{M}{W_{xu}} \text{ hvor } W_{xu} = \frac{I_x}{y_u}$$



**Arealmoment**

Annet arealmoment

$$I_x = \int y^2 dA, \text{ om x-aksen}$$

$$I_y = \int x^2 dA, \text{ om y-aksen}$$

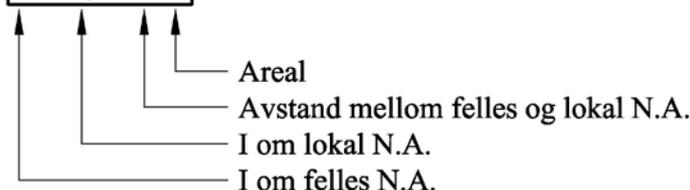
Arealmoment

$$S_x = \int y dA = \Sigma y_i A_i$$

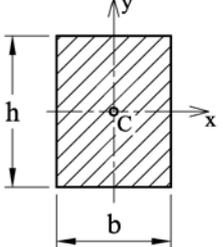
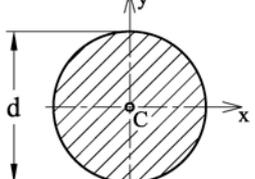
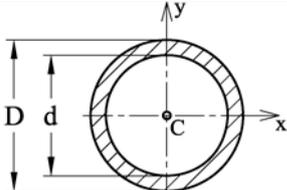
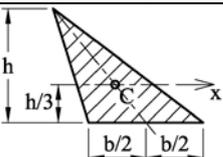
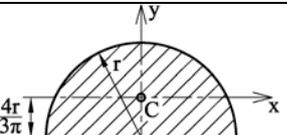
$$S_y = \int x dA = \Sigma x_i A_i$$

Steiners formel:

$$I = I_o + d^2 A$$

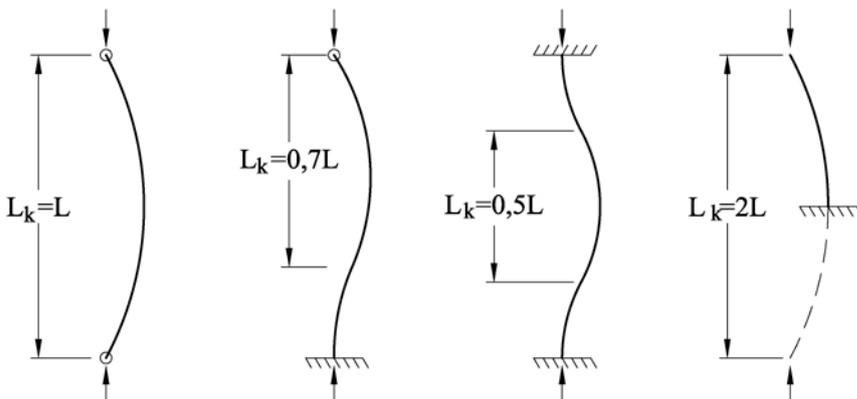


**Tverrsnittsegenskaper**

Snittflate	Annet arealmoment	Motstandsmoment	Polart arealmoment
	$I_x = \frac{bh^3}{12}$ $I_y = \frac{hb^3}{12}$	$W_x = \frac{bh^2}{6}$ $W_y = \frac{hb^2}{6}$	$I_P = \frac{bh}{12}(h^2 + b^2)$
	$I_x = I_y = \frac{\pi d^4}{64}$	$W_x = W_y = \frac{\pi d^3}{32}$	$I_P = \frac{\pi d^4}{32}$
	$I_x = I_y = \frac{\pi}{64}(D^4 - d^4)$	$W_x = W_y = \frac{\pi}{32} \cdot \frac{D^4 - d^4}{D}$	$I_P = \frac{\pi}{32}(D^4 - d^4)$
	$I_x = \frac{bh^3}{36}$		
	$I_x = 0,12r^4$ $I_y = \frac{\pi}{8}r^4$		

**Knekking**

Knekk lengde:



Elastisk knekking (Eulerlast):  $F_k = \frac{\pi^2 EI_0}{L_k^2}$  alternativt  $\sigma_k = \frac{\pi^2 E}{\lambda}$

Treghetsradius:  $i = \sqrt{\frac{I_0}{A}}$  Slankhet:  $\lambda = \frac{L_k}{i}$

Tetmajers formler for plastisk knekning:

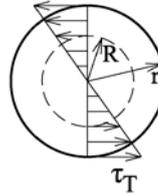
$$\text{St37:} \quad \sigma_k = 310 - 1,14\lambda \quad \text{for} \quad \lambda \in <10, 105>$$

$$\text{St50/St60:} \quad \sigma_k = 335 - 0,62\lambda \quad \text{for} \quad \lambda \in <10, 89>$$

### Torsjon

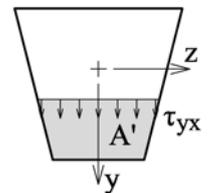
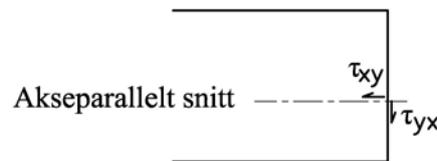
Torsjonsskjærspenning i sirkulærsylindrisk tverrsnitt

$$\tau_T = \frac{M_T}{I_p} R \quad \text{hvor} \quad I_p = \int R^2 dA$$



### Bøyeindusert skjærspenning

$$\tau_{yx} = \tau_{xy} = \frac{V}{Ib} S' \quad \text{hvor} \quad S' = \int_{A'} y dA = \Sigma y_i A_i$$



Rektangulært tverrsnitt

$$\tau_{\max} = 1,5 \frac{V}{A}$$

Sirkulært tverrsnitt

$$\tau_{\max} = 1,33 \frac{V}{A}$$

Tynnvegget rør

$$\tau_{\max} = 2,0 \frac{V}{A}$$

### Sikkerhetsfaktor

$$n = \frac{\text{konstruksjonens teoretiske kapasitet}}{\text{konstruksjonens største (tillatte) påkjenning}}$$

### Dimensjoneringskriterium for flytning (von Mises)

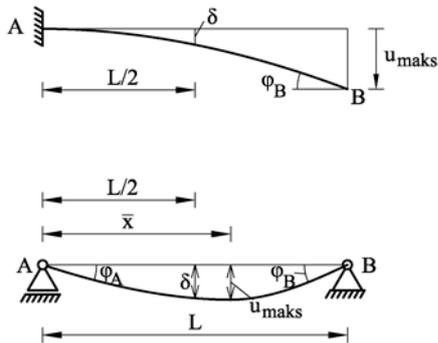
Jevnføringsspenning:  $\sigma_j < \frac{R_e}{n}$ , hvor  $R_e$  = Flytegrense

Kun normalspenning:  $\sigma_j = \sigma_x$

En-akset spenningstilstand:  $\sigma_j = \sqrt{\sigma_x^2 + 3\tau_{yx}^2}$

Plan spenningstilstand:  $\sigma_j = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{yx}^2}$

**Deformasjon av enkle bjelker**



$EI$  = konstant

$\delta$  = utbøyning på midten

$u_{maks}$  = maksimal utbøyning

$u_{last}$  = utbøyning under punktlast

$\bar{x}$  = avstand fra A til punkt med maksimal utbøyning

$\varphi_A$  = tangenthelning ved A

$\varphi_B$  = tangenthelning ved B

1		$u_{maks} = \frac{qL^4}{8EI}, \varphi_B = \frac{qL^3}{6EI}, \delta = \frac{17}{384} \frac{qL^4}{EI}$
2		$u_{maks} = \frac{FL^3}{3EI}, \varphi_B = \frac{FL^2}{2EI}, \delta = \frac{5}{48} \frac{FL^3}{EI}$
3		$u_{maks} = \frac{ML^2}{2EI}, \varphi_B = \frac{ML}{EI}$
4		$u_{maks} = \frac{ML^2}{9\sqrt{3}EI}, \bar{x} = (1 - \frac{\sqrt{3}}{3})L, \varphi_A = \frac{ML}{3EI}, \varphi_B = \frac{ML}{6EI}, \delta = \frac{ML^2}{16EI}$
5		$u_{maks} = \frac{Fb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}, \bar{x} = \sqrt{\frac{L^2 - b^2}{3}}, u_{last} = \frac{Fa^2b^2}{3LEI}$ $\varphi_A = \frac{F \cdot ab(a + 2b)}{6LEI}, \varphi_B = \frac{F \cdot ab(b + 2a)}{6LEI}, \delta = \frac{Fb(3L^2 - 4b^2)}{48EI}$
6		$\delta = u_{maks} = \frac{FL^3}{48EI}, \varphi_A = \varphi_B = \frac{FL^2}{16EI}$
7		$\delta = u_{maks} = \frac{5}{384} \frac{qL^4}{EI}, \varphi_A = \varphi_B = \frac{qL^3}{24EI}$